

Marking Scheme
Applied Mathematics
Term - I
Code-241

Q.N.	Correct option	Hints/Solutions
Section – A		
1	c	$5 \odot_8 11 = (5 \times 11) \bmod 8 = 55 \bmod 8 = 7$
2	a	For distinct $x, y > 0$; $AM > GM \Rightarrow \frac{x+y}{2} > \sqrt{xy} \Rightarrow x + y > 2\sqrt{xy}$
3	c	Let x be the speed of the stream $\therefore 8 + x = 3(8 - x) \Rightarrow 4x = 16 \Rightarrow x = 4 \text{ km/h}$
4	d	Since $3 (x + 4)$ is true for $x = 35$
5	d	$ \text{adj}(A) = A ^{n-1} \Rightarrow \text{adj}(A) = (-2)^2 = 4$
6	a	The summation of product of a_{ij} of 2 nd column with corresponding c_{ij} of 3 column = 0
7	c	$ AB = 12 \Rightarrow A B = 12$ $\Rightarrow -4 A = 12 \Rightarrow A = -3$
8	a	If $\Delta = 0$ and at least (one of $\Delta_x, \Delta_y, \Delta_z$) $\neq 0$ The system of linear equations has no solution
9	c	$C(x) = x^2 + 30x + 1500$ $MC = C'(x) = 2x + 30$ MC when 10 units are produced = $C'(10) = ₹ 50$
10	c	$y = \frac{1}{x} \Rightarrow \frac{dy}{dx} = -\frac{1}{x^2} < 0$ for $(-\infty, 0)$ and $(0, \infty)$
11	b	$y = x^3 + x \Rightarrow \frac{dy}{dx} = 3x^2 + 1 \Rightarrow \left(\frac{dy}{dx}\right)_{x=1} = 4$ \therefore Equation to target is $y - 2 = 4(x - 1) \Rightarrow 4x - y = 2$
12	b	Expected number of votes = $np = \frac{70}{100} \times 120000 = 84000$
13	d	The total area under the normal distribution curve above the base line is 1
14	c	$\sum p_i = 1 \Rightarrow 7k = 1 \Rightarrow k = \frac{1}{7}$ Now, $P(x \geq 3) = 3k = \frac{3}{7}$
15	b	For Poisson distribution Mean = variance = $np = 20000 \times \frac{1}{10000} = 2$
16	d	$\sum_{k=0}^{\infty} \frac{e^{-\lambda} \lambda^k}{k!} = \text{Total probability} = 1$
17	b	$p = 0.05 = \frac{1}{20}, q = \frac{19}{20}$ $P(x \geq 1) = 1 - P(0) = 1 - {}_6C_0 \left(\frac{1}{20}\right)^0 \left(\frac{19}{20}\right)^6 = 1 - \left(\frac{19}{20}\right)^6$
18	c	In Laspeyre's price index the weight are taken as base year quantities
19	a	$P_{01}^P = \frac{\sum p_1 q_1}{\sum p_0 q_1} \times 100 = \frac{506}{451} \times 100 = 112.19$
20	c	Marshall- Edgeworth formula uses the arithmetic mean of the base and current year quantities.

Section –B

21	c	Since Vijay is faster by 4 secs. ∴ he beats Samuel by $= \frac{100}{16} \times 4 = 25 \text{ meters}$									
22	b	∴ $876 \pmod{24} = 12$ ∴ 8.40 PM will change to 8.40 AM after 12 hours, further after 30 minutes the time will be 9.10 AM									
23	b	Let total capital be $= x$ & let C's contribution $= y$, B's contribution $= \frac{x}{3}$, A's Contribution $= \frac{x}{3} + y$. Now (A+B+C)'s contribution $= x \Rightarrow x = 6y$ <i>hence their contributions are $2y : y : 2y$ i.e., in the ratio 3:2:1</i>									
24	d	The relation R_m defined as $a \equiv b \pmod{m}$ is reflexive, symmetric and transitive ∴ R_m is an equivalent relation									
25	b	Time ratio $= 2 : 3 : 4$ Profit sharing ratio $= 6 : 7 : 8$ Investment ratio $= \frac{6}{2} : \frac{7}{3} : \frac{8}{4} \left(\frac{\text{Profit}}{\text{Time}} \right)$ $= 9 : 7 : 6$									
26	c	$2a + b + c - 3d = b + c \quad (\because a = d = 0)$ $= b + (-b) \quad (\because c = -b)$ $= 0$									
27	d	∴ $1 - a_{11}, 1 - a_{22} > 0$ and $ I - A > 0$ and it is true only for $\begin{pmatrix} 0.3 & 0.2 \\ 0.1 & 0.5 \end{pmatrix}$									
28	c	$y = x $ has a sharp point at $x = 0$ $y = x $ is continuous but not differentiable at $x = 0$									
29	a	$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{2a}{2at} = \frac{1}{t} \Rightarrow \frac{d^2y}{dx^2} = -\frac{1}{t^2} \times \frac{dt}{dx} = -\frac{1}{2at^3}$									
30	c	$TC = VC + FC = x^2 + 2x + 10000$ $AC = x + 2 + \frac{10000}{x}$ $\frac{d(AC)}{dx} = 1 - \frac{10000}{x^2} = 0 \Rightarrow x = 100$									
31	a	<table border="0" style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th>Prize (x_i)</th> <th>p_i</th> <th>$x_i p_i$</th> </tr> </thead> <tbody> <tr> <td>500000</td> <td>$\frac{1}{10000}$</td> <td>50</td> </tr> <tr> <td>0</td> <td>$\frac{9999}{10000}$</td> <td>0</td> </tr> </tbody> </table> <p>So, $\sum x_i p_i = 50$ Net expected gain $= 50 - 100 = -50$ So gain is -50</p>	Prize (x_i)	p_i	$x_i p_i$	500000	$\frac{1}{10000}$	50	0	$\frac{9999}{10000}$	0
Prize (x_i)	p_i	$x_i p_i$									
500000	$\frac{1}{10000}$	50									
0	$\frac{9999}{10000}$	0									
32	c	$P(r < 2) = P(0 \text{ or } 1) = 10C_0 \left(\frac{1}{2}\right)^{10} + 10C_1 \left(\frac{1}{2}\right)^{10} = \frac{1+10}{1024} = \frac{11}{1024}$									
33	d	$n = 100, p = \frac{1}{10}, q = \frac{9}{10}$ $\sigma = \sqrt{npq} = \sqrt{100 \times \frac{1}{10} \times \frac{9}{10}} = 3$									
34	a	$P(x > 518) = 1 - P(x < 518)$ $= 1 - P(z < 1) = 1 - 0.8413$ $= 0.1587$									
35	b	$P(x < 54) = P(z < 1.5)$ $= 0.9332$ $= 93.32 \%$									

36	b	$\frac{\sum P_1}{\sum P_0} \times 100 = \frac{340}{300} = 113.34$
37	b	$P_{01}^F = \sqrt{(P_{01}^L \times P_{01}^P)} = \sqrt{118.4 \times 117.5} = 117.95$
38	c	Since, $L:P = 28:27$, $\therefore \frac{\sum p_1 q_0}{\sum p_0 q_0} \times \frac{\sum p_0 q_1}{\sum p_1 q_1} = \frac{28}{27}$ $\Rightarrow 9x + 36 = 40 + 8x \Rightarrow x = 4$
39	a	$\frac{\sum (\frac{p_1}{p_0})(p_0 q_0)}{\sum (p_0 q_0)} \times 100$
40	d	Time reversal Test is satisfied by Fishers ideal index
41	a	$C = -5\%$ $d = 10\%$ $m = 7\%$ $(d - m) : (m - c) = 1 : 4$ Quantity sold at 10 % profit = $\frac{4}{5} \times 250 = 200$ Kg
42	d	Portion of cistern filled by both pipes in 1 hour = $\frac{1}{8} + \frac{1}{12} = \frac{5}{24}$. Time taken by both pipes to fill the cistern = 4 h 48 mints Time taken to fill tank due to leakage = 5 h Work done by leakage in 1 h = $\frac{5}{24} - \frac{1}{5} = \frac{1}{120}$ Time taken by leakage to empty the tank = 120 h
43	a	$TR = px = \frac{75x - x^2}{3}$ $P = TR - TC = \frac{75x - x^2}{3} - (3x + 100)$ $\frac{dP}{dx} = 22 - \frac{2}{3}x = 0 \Rightarrow x = 33$
44	d	$P(X \geq 1) = 1 - P(0) = 1 - \frac{e^{-2}(2)^0}{0!}$ $= 1 - e^{-2} = 0.8647$
45	c	$P(10 < x < 30)$ $= P(-2.5 < Z < 2.5)$ $= P(z < 2.5) - P(z < -2.5)$ $= 0.9876$
46	b	Since elements of technology matrix a_{ij} , represents units of sector i to produce 1 unit of sector j $\therefore \begin{pmatrix} 0.50 & 0.25 \\ 0.10 & 0.25 \end{pmatrix}$ is the technology matrix
47	c	$I - A = \begin{pmatrix} 0.50 & -0.25 \\ -0.10 & 0.75 \end{pmatrix} \Rightarrow (I - A)^{-1} = \frac{20}{7} \begin{pmatrix} 0.75 & 0.25 \\ 0.1 & 0.5 \end{pmatrix}$ $= \frac{1}{7} \begin{pmatrix} 15 & 5 \\ 2 & 10 \end{pmatrix}$
48	b	System is viable if $ I - A > 0$ and $1 - a_{11} > 0, 1 - a_{22} > 0$
49	a	$X = (I - A)^{-1}D = \frac{1}{7} \begin{pmatrix} 15 & 5 \\ 2 & 10 \end{pmatrix} \begin{pmatrix} 7000 \\ 14000 \end{pmatrix} = \begin{pmatrix} 25000 \\ 22000 \end{pmatrix}$
50	d	Internal consumption = total production - external demand $= \begin{pmatrix} 25000 \\ 22000 \end{pmatrix} - \begin{pmatrix} 7000 \\ 14000 \end{pmatrix} = \begin{pmatrix} 18000 \\ 8000 \end{pmatrix}$