

Mathematics
Sample Question Paper
Class XII

Class:12
Time 3hrs

Max Mks:100
No of pages:4

General Instructions:

- All questions are compulsory.
- The question paper consists of 29 questions divided into three sections - A, B and C.
- Section - A comprises of 10 questions of one mark each.
- Section - B is of 12 questions of four marks each.
- Section - C comprises of 7 questions of six marks each.
- Internal choice has been provided in four marks question and six marks question.
- You have to attempt any one of the alternatives in all such questions .
- Use of calculator not permitted.

SECTION A

1. Compute $\text{adj}(A)$, where $A = \begin{bmatrix} 3 & -4 \\ 5 & 7 \end{bmatrix}$.
2. Evaluate $\sin(\tan^{-1}(x))$, $|x| < 1$.
3. Write the position vector of the mid-point of the vector joining the points $p(2,3,4)$ and $q(4,-1,2)$.
4. Write the distance from following plane from the origin $2x-y+2z+1 = 0$.
5. If A is irreversible matrix of order 5 and $|A| = 5$, then find $[\text{adj},A]$.
6. If $A^{-1} = \begin{bmatrix} 3 & 4 \\ -2 & 8 \end{bmatrix}$, find $(A^T)^{-1}$.

7. Solve for x , $\begin{vmatrix} x & 3 \\ 5 & 2x \end{vmatrix} = \begin{vmatrix} 5 & -4 \\ 5 & 3 \end{vmatrix}$.

8. Find a vector in the direction of $\vec{a} = 6\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$ whose magnitude is 5.

9. Find the value of C s.t. $A - B + 2C = 0$, where

$$A = \begin{bmatrix} 1 & 3 & 4 \\ 2 & -1 & -3 \end{bmatrix}, B = A = \begin{bmatrix} -2 & 5 & -1 \\ 3 & 0 & 8 \end{bmatrix}$$

10. Evaluate $\int \left(\frac{\sin 2x}{3 + \sec x} \right) dx$

SECTION B

11. Use mathematical induction prove that $\frac{dy}{dx}(x^n) = nx^{(n-1)}$ for all n belongs \mathbb{N} for all positive integers n .

12. Show that an onto function $f : \{1, 2, 3\} \rightarrow \{1, 2, 3\}$ is always 1-1.

13. Express the following matrix as the sum of a symmetric and skew-symmetric matrix:

$$\begin{bmatrix} 7 & 1 & 0 \\ 1 & -5 & 6 \\ 0 & 8 & -4 \end{bmatrix}$$

14. Find the derivative of the function given by $f(x) = (1+x)(1+x^2)(1+x^4)(1+x^8)$ and hence find $f'(3)$.

15. Find the equation of the plane passing through the points $(2,3,4), (5,6,7), (1,0,0)$.

16. Find the value of the constant S s.t. the scalar product of the vector $\mathbf{i} + \mathbf{j} + \mathbf{k}$ with the unit vector parallel to the sum of the vectors $2\mathbf{i} + 4\mathbf{j} - 5\mathbf{k}$ and $5\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$ is equal to 1.

17. Using properties of determinants, prove that

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(a+b)(b+c)$$

18. Find the equation of the plane through the line of intersection of the planes $x+y+z=1$ and $2x+3y+4z=5$, which is perpendicular to the plane $x-y+z=0$.
19. Express the following matrix as the sum of symmetric and skew symmetric matrix, and verify your answer.

$$\begin{vmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{vmatrix}$$

20. Integrate $\int x^{2n-1} \sin x^n dx$.

21. Show that $y = \log(1+x) - \frac{2x}{2+x}, x > -1$ is an increasing function of x throughout its domain.

22. On multiplying choice examination with three possible answers for each of the five questions, what is the probability that a candidate would get four or more correct answer just by guessing?

SECTION C

23. Kellogs is a new cereal formed of a mixture of bran and rice that contains at least 88 grams of protein and at least 36 mg of iron. Knowing that bran contains 80 grams of protein and 40 mg of iron per kg and rice contains 100 grams of protein and 30 mg of iron per kg, find the minimum cost of producing this cereal if bran costs Rs. 5 per kg and rice costs Rs. 4 per kg.

24. Solve the following system of equations:

$$x + y + z = 6$$

$$x + 2y + 3z = 14$$

$$x + 4y + 7z = 30$$

or

Using matrix method solve the following system of equations:

$$2x + y + z = 3$$

$$3x - y + z = 0$$

$$x - 2y + 3z = -6$$

25. Find the area of the region bounded by $y^2 = 4x$, $x = 1$, $x = 4$ and x -axis in the first quadrant

or

Evaluate $\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^n (x_k^2 + x_k + 1)$ as limit of a sum.

26. Find the area of the smaller region enclosed between the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and the line $\frac{x}{a} + \frac{y}{b} = 1$

27. If $y = \sqrt{\log x + \sqrt{\log x + \sqrt{\log x + \dots \infty}}}$, show that $(2y - 1) \frac{dy}{dx} = \frac{1}{x}$

28. Sketch the graph of $y = |x + 5|$ and evaluate the area under the curve $y = x + 5$ above x-axis and between $x = -7$ to $x = 0$.

29. Show that the semi vertical angle of the right circular cone of given total surface and

maximum volume is $\sin^{-1} \left(\frac{1}{3} \right)$