MATHEMATICS – Code No. 041
MARKING SCHEME
CLASS – XII (2025-26)

SECTION-A (MCQs of 1 mark each)		
Sol. N.	**Hint / Solution**	**Marks**
1	Clearly from the graph Domain is $\left[-\frac{1}{2},\frac{1}{2}\right]$ So graph is of the function $\sin^{-1}(2x)$ **Answer is (B)** $\sin^{-1}(2x)$	**1**
1 (V.I.)	Domain is $\left[-\frac{1}{3},\frac{1}{3}\right]$ So the function is $\cos^{-1}(3x)$ **Answer is (C)** $\cos^{-1}(3x)$	**1**
2	AB is defined so n=4 AC is defined so p=4 AB and AC are square matrices of same order so $m \times 3 = m \times q \Rightarrow q = 3 = m$ **Answer is (A)** $m = q = 3 \quad and\, n = p = 4$	**1**
3	As A is skew symmetric So $p = 0, q = 2, r = -3, t = 4$ So $\frac{q+t}{p+r} = \frac{6}{-3} = -2$ **Answer is (A) -2**	**1**
4	$\|adj\ A\| = 27 \Rightarrow \|A\|^3 = 27 = 3^3 \Rightarrow \|A\| = 3$ $A\,(adj\ A) = \|A\|\, I = 3\, I$ **Answer is (C) 3 $\boldsymbol{I}$**	**1**
5	Inverse of the matrix $\begin{bmatrix}3&0&0\\0&2&0\\0&0&5\end{bmatrix} = \begin{bmatrix}\frac{1}{3}&0&0\\0&\frac{1}{2}&0\\0&0&\frac{1}{5}\end{bmatrix}$ **Answer is (B)**	**1**
6	$\begin{vmatrix}\cos 67^o & \sin 67^o\\ \sin 23^o & \cos 23^o\end{vmatrix} = \cos 67^o \cos 23^o - \sin 67^o \sin 23^o = \cos(67^o + 23^o) =$ $\cos 90^o = 0$ **Answer is (A) 0**	**1**
7	$f(x)$ is continuous at $x = \pi$ $\Rightarrow \lim\limits_{x\to\pi^-}(kx+1) = \lim\limits_{x\to\pi^+} cos\ x = f(\pi)$ $\Rightarrow \lim\limits_{h\to 0}[k(\pi - h) + 1] = \lim\limits_{h\to 0} cos(\pi + h) = k\pi + 1$ $\Rightarrow k\pi + 1 = -1 \qquad \Rightarrow k = \frac{-2}{\pi}$ **Answer is (D)** $\boldsymbol{\frac{-2}{\pi}}$	**1**

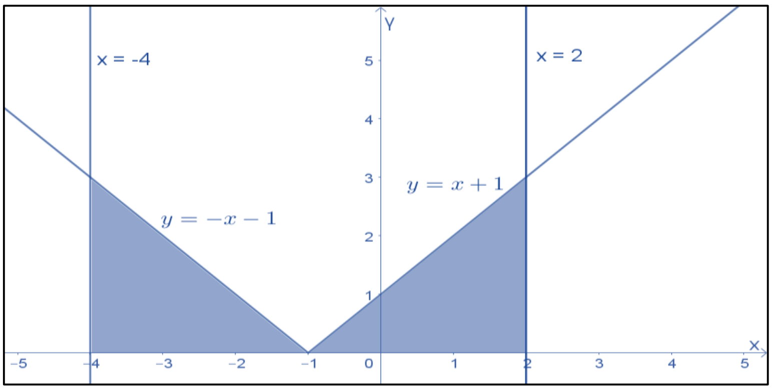
8	$f(x) = x\tan^{-1}x$ $f'(x) = 1.\tan^{-1}x + x.\frac{1}{1+x^2}$ $f'(1) = 1.\tan^{-1}1 + \frac{1}{1+1} = \frac{\pi}{4} + \frac{1}{2}$ **Answer is (B) $\frac{\pi}{4} + \frac{1}{2}$**	**1**
9	$f(x) = 10 - x - 2x^2$ $\Rightarrow f'(x) = -1 - 4x$ For increasing function $f'(x) \geq 0$ $\Rightarrow -(1+4x) \geq 0$ $\Rightarrow (1+4x) \leq 0$ $\Rightarrow x \leq -1/4$ $\Rightarrow x \in \left(-\infty, -\frac{1}{4}\right]$ **Answer is (A) $\left(-\infty, -\frac{1}{4}\right]$**	**1**
10	$xdx + ydy = 0$ $\Rightarrow \int xdx = -\int ydy$ $\Rightarrow \frac{x^2}{2} = -\frac{y^2}{2} + k$ $\Rightarrow x^2 + y^2 = 2k$ Solution is $x^2 + y^2 = 2k, k$ being an arbitrary constant. **Answer is (C) Circles**	**1**
11	$I = \int_a^b x f(x)dx = \int_a^b (a+b-x)f(a+b-x)dx$ $\Rightarrow I = \int_a^b (a+b-x)f(x)dx$ (given $f(a+b-x) = f(x)$) $\Rightarrow I = \int_a^b (a+b) f(x)dx - \int_a^b x f(x)dx$ $\Rightarrow 2I = (a+b)\int_a^b f(x)dx$ $\Rightarrow I = \frac{1}{2}(a+b)\int_a^b f(x)dx$ **Answer is (D) $\frac{a+b}{2}\int_a^b f(x)dx$**	**1**
12	Let $I = \int x^3 sin^4(x^4)\cos(x^4)\,dx$ Let $\sin(x^4) = t \Rightarrow 4x^3\cos(x^4)\,dx = dt \Rightarrow x^3\cos(x^4) = \frac{1}{4}dt$ Thus $I = \int t^4\left(\frac{1}{4}dt\right) = \frac{1}{20}t^5 + C = \frac{1}{20} sin^5(x^4) + C$ $\Rightarrow I = \frac{1}{20} sin^5(x^4) + C = a\ sin^5(x^4) + C$ So, $a = \frac{1}{20}$ **Answer is (B) $\frac{1}{20}$**	**1**
13	The projection of the vector $\hat{\imath} + 2\hat{\jmath} + \hat{k}$ on the line $\vec{r} = (3\hat{\imath} - \hat{\jmath}) + \lambda(\hat{\imath} + 2\hat{\jmath} + 3\hat{k})$ is $\frac{1x1 + 2x2 + 1x3}{\sqrt{1^2+2^2+3^2}} = \frac{8}{\sqrt{14}}$ units **Answer is (C) $\frac{8}{\sqrt{14}}$ units**	**1**

14	The distance of the point (a, b, c) from the y-axis is $\sqrt{a^2+c^2}$ So, the distance is $\sqrt{3^2+5^2}=\sqrt{34}$ units. **Answer is (B) $\sqrt{34}$ units**	**1**
15	$(2\vec{a}.\hat{\imath})\hat{\imath}-(\vec{b}.\hat{\jmath})\hat{\jmath}+(\vec{c}.\hat{k})\hat{k}=(2\times 3)\hat{\imath}-(1)\hat{\jmath}+(2)\hat{k}$ $=6\hat{\imath}-\hat{\jmath}+2\hat{k}=\vec{c}$ **Answer is (D) $\vec{c}$**	**1**
16	The points (1,0) and (0,2) satisfy the equation $2x+y=2$ And shaded region shows that (0,0) doesn't lie in the feasible solution region So, the inequality is $2x+y\geq 2$ **Answer is (B) $2x+y\geq 2$**	**1**
16 (V.I.)	(4,0) and (0,3) gives maximum value so $Z_{(4,0)}=Z_{(0,3)}\Rightarrow 4a+c=3b+c\Rightarrow 4a=3b$ **Answer is (A) $4a=3b$**	**1**
17	The student may read the point (2,9) from the line on the graph. The student may find the equation $3x+y=15$ joining (5,0) and (0,15) and then verify the point (2,9) satisfies it. **Answer is (A) (2,9)**	**1**
17 (V.I.)	**Answer is (C)** Open Half plane that contains origin, but not the points of the line $3x+5y=10$	**1**
18	**Answer is (B) $\frac{1}{100}$** The person knows the first 4 digits. So the person has to guess the remaining two digits. P (guessing the PIN)=1×1×1×1×$\frac{1}{10}\times\frac{1}{10}=\frac{1}{100}$	**1**
19	$\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)+\tan^{-1}1-\sec^{-1}(\sqrt{2})=\frac{\pi}{3}+\frac{\pi}{4}-\frac{\pi}{4}=\frac{\pi}{3}\neq\frac{\pi}{4}$ So, A is false. Principal Value branch of $\sin^{-1}x$ is $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ and that of $sec^{-1}x$ is $[0,\pi]-\left\{\frac{\pi}{2}\right\}$. So, R is true **Answer is (D)Assertion is false, but Reason is true**	**1**
20	C. $\vec{r}\times(\vec{a}+\vec{b})=\vec{0}\Rightarrow\vec{r}$ is parallel to $(\vec{a}+\vec{b})$ and $(\vec{a}+\vec{b})$ lies on the plane of $\vec{a}$ and $\vec{b}$. So, $\vec{r}$ is parallel to the plane of $\vec{a}$ and $\vec{b}\Rightarrow\vec{r}$ is perpendicular to $(\vec{a}\times\vec{b})$. So, Assertion is true But $(\vec{a}+\vec{b})$ lies on the plane of $\vec{a}$ and $\vec{b}$, so $(\vec{a}+\vec{b})$ is not perpendicular to the plane of $\vec{a}$ *and* $\vec{b}$ Therefore, Reason is false. **Answer is (C) Assertion is true, but Reason is false**	**1**

SECTION B
(VSA type questions of 2 marks each)

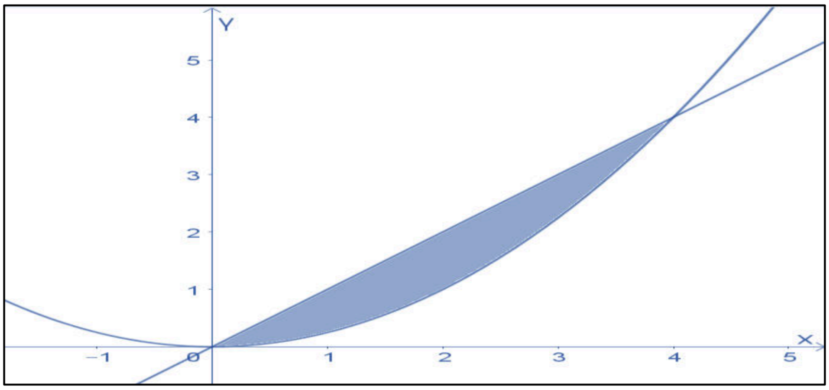
21A	$\tan\left(\tan^{-1}(-1)+\frac{\pi}{3}\right)=\tan\left(-\frac{\pi}{4}+\frac{\pi}{3}\right)$	½
	$=\frac{\tan\frac{\pi}{3}-\tan\frac{\pi}{4}}{1+\tan\frac{\pi}{3}\tan\frac{\pi}{4}}$	1
	$=\frac{\sqrt{3}-1}{1+\sqrt{3}}$ *or* $2-\sqrt{3}$	½
	OR	OR
21B	For domain, $-1\le 3x-2\le 1$	½
	$\Rightarrow 1\le 3x\le 3$	½
	$\Rightarrow \frac{1}{3}\le x\le 1$	½
	So, domain of $\cos^{-1}(3x-2)$ is $\left[\frac{1}{3},1\right]$	½
22	$y=\log\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)$	
	Differentiating with respect to x	
	$\frac{dy}{dx}=\frac{1}{\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)}.sec^2\left(\frac{\pi}{4}+\frac{x}{2}\right).\frac{1}{2}$	½
	$=\frac{\cos\left(\frac{\pi}{4}+\frac{x}{2}\right)}{\sin\left(\frac{\pi}{4}+\frac{x}{2}\right)}.\frac{1}{cos^2\left(\frac{\pi}{4}+\frac{x}{2}\right)}.\frac{1}{2}$	
	$=\frac{1}{2\sin\left(\frac{\pi}{4}+\frac{x}{2}\right)\cos\left(\frac{\pi}{4}+\frac{x}{2}\right)}=\frac{1}{\sin\left(\frac{\pi}{2}+x\right)}=\frac{1}{\cos x}$	1
	$\Longrightarrow \frac{dy}{dx}-\sec x=0$	½
23A	$\int\frac{(x-3)e^x}{(x-1)^3}\,dx=\int\frac{(x-1-2)e^x}{(x-1)^3}\,dx$	
	$=\int\left(\frac{1}{(x-1)^2}-\frac{2}{(x-1)^3}\right)e^x\,dx=\int\left(\frac{1}{(x-1)^2}+\frac{d}{dx}\left(\frac{1}{(x-1)^2}\right)\right)e^x\,dx$	1
	$=\frac{e^x}{(x-1)^2}$ + c (as $\int(f(x)+f'(x))e^x\,dx=e^x\,f(x)$ + c)	1
	OR	OR
23B	A = $\int_0^4 x\,dy$ = $\int_0^4\sqrt{y}\,dy$	1
	= $\frac{2}{3}\times y^{3/2}\Big]_{y=0}^{y=4}$ = $\frac{16}{3}$ sq. units	1
23B	**For Visually Impaired:**	
	A = $\int_0^3 y\,dx$ = $\int_0^3\sqrt{x}\,dx$	1
	= $\frac{2}{3}\times x^{3/2}\Big]_{x=0}^{x=3}$ = $2\sqrt{3}$ sq. units	1

24	Given $f(x+y) = f(x)f(y)$ $f(0+5) = f(0)f(5)$ $\Rightarrow \quad f(0) = 1$ $f'(5) = \lim_{h\to 0}\frac{f(5+h)-f(5)}{h} = \lim_{h\to 0}\frac{f(5)f(h)-f(5)}{h}$ $[\because f(x+y) = f(x)f(y)]$ $= \lim_{h\to 0}\frac{2f(h)-2}{h}$ $[\because f(5) = 2]$ $= 2\lim_{h\to 0}\frac{f(h)-1}{h} = 2\lim_{h\to 0}\frac{f(h)-f(0)}{h} = 2f'(0)$ $= 2(3)$ $[\because f'(0) = 3]$ $= 6$	½ 1 ½
25	The vector $\overrightarrow{OP} = \frac{1}{2}(4\hat{\imath} + 4\hat{k}) = 2\hat{\imath} + 2\hat{k}$ Area of the parallelogram formed by the two adjacent sides as OA and OP $= \lvert(\overrightarrow{OA} \times \overrightarrow{OP})\rvert = \left\lVert\begin{matrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ 1 & 1 & 1\\ 2 & 0 & 2\end{matrix}\right\rVert$ $= \lvert 2\hat{\imath} - 2\hat{k}\rvert$ $= 2\sqrt{2}$ square units.	½ ½ ½ ½
SECTION C **(SA type questions of 3 marks each)**		
26A	$x^y = e^{x-y}$ Taking log of both sides $y\log x = (x-y)\log e$ $y\log x$ + y = x (since $\log e = 1$) $\Rightarrow \quad y = \frac{x}{1+\log x}$ Differentiating with respect to x $\frac{dy}{dx} = \frac{(1+\log x).1 - x.\frac{1}{x}}{(1+\log x)^2}$ $= \frac{\log x}{(\log e + \log x)^2}$ $= \frac{\log x}{(\log(x\,e))^2}$ Now $\left.\frac{dy}{dx}\right]_{x=e} = \frac{\log e}{(\log e^2)^2} = \frac{1}{(2\log e)^2} = \frac{1}{2^2} = \frac{1}{4}$ (as $\log e = 1$) **Alternative Solution:** $x^y = e^{x-y}$ Taking log of both sides $y\log x = (x-y)\log e$ $y\log x$ + y = x (since $\log e = 1$) Differentiating both sides w.r.t. x $\log x\frac{dy}{dx} + \frac{y}{x} + \frac{dy}{dx} = 1$ $\Rightarrow \frac{dy}{dx}(1+\log x) = 1 - \frac{y}{x}$ $\Rightarrow \frac{dy}{dx} = \frac{x-y}{x(1+\log x)} = \frac{x - \frac{x}{1+\log x}}{x(1+\log x)} = \frac{x(1+\log x)-x}{x(1+\log x)^2} = \frac{x(1+\log x - 1)}{x(\log e + \log x)^2} = \frac{\log x}{(\log(xe))^2}$ Now $\left.\frac{dy}{dx}\right]_{x=e} = \frac{\log e}{(\log e^2)^2} = \frac{1}{(2\log e)^2} = \frac{1}{2^2} = \frac{1}{4}$ (as $\log e = 1$)	1 1 1

	OR	OR
26B	$\frac{dx}{d\theta} = a(1 - \cos\theta)$, $\frac{dy}{d\theta} = a(0 + \sin\theta)$, $\Rightarrow \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{a\sin\theta}{a(1-\cos\theta)}$ $= \frac{2\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)}{2sin^2\left(\frac{\theta}{2}\right)} = \cot\frac{\theta}{2}$ $\Rightarrow \frac{d^2y}{dx^2} = -\frac{1}{2}\,cosec^2\left(\frac{\theta}{2}\right)\frac{d\theta}{dx}$ $= -\frac{1}{2a}\,cosec^2\left(\frac{\theta}{2}\right)\frac{1}{2sin^2\left(\frac{\theta}{2}\right)}$ $= -\frac{1}{4a}cosec^4\left(\frac{\theta}{2}\right)$	1 1 1
27	Let r be the radius of ice ball at time t . $V = \frac{4}{3}\pi r^3$………… (1) S = $4\pi r^2$…………..(2) Given $\frac{dV}{dt} \propto$ S $\Longrightarrow \frac{dV}{dt} = -$ k S (where k is some positive constant) ………(3) Differentiating (1) w.r.t. t, we get $\frac{dV}{dt} = \frac{4}{3}\pi.(3r^2)\frac{dr}{dt}$ $\frac{dV}{dt} = 4\pi r^2\frac{dr}{dt}$ ………. (4) $\Longrightarrow$ - k S = $4\pi r^2\frac{dr}{dt}$ (from (3) and (4)) $\Longrightarrow$ - k S = S $\frac{dr}{dt}$ (using (2)) $\Longrightarrow \frac{dr}{dt} = -$ k $\Rightarrow$ radius of the ice-ball decreases at a constant rate	½ ½ 1 ½ ½
28A	 $\int_{-4}^{2}\|x+1\|dx = \int_{-4}^{-1}(-x-1)\,dx + \int_{-1}^{2}(x+1)\,dx$ $= -\frac{(x+1)^2}{2}\Big]_{-4}^{-1} + \frac{(x+1)^2}{2}\Big]_{-1}^{2}$ $= -\left(0 - \frac{9}{2}\right) + \left(\frac{9}{2} - 0\right) = 9$ It represent the area of shaded region bounded by the curve $y = \|x+1\|$, $x -$ axis and the lines $x = -4$ and $x = 2$	1 ½ ½ ½ ½

OR

28B



1

Required Area = $\int_0^4 x\,dx - \int_0^4 \frac{x^2}{4}\,dx$ 1

$= \frac{x^2}{2}\Big]_0^4 - \frac{1}{12}[x^3]_0^4$ ½

$= \frac{1}{2}(16-0) - \frac{1}{12}(64-0) = 8 - \frac{16}{3} = \frac{8}{3}$ sq. units ½

For Visually Impaired:

$y = |x+1| = f(x) = \begin{cases} -x-1, & x < -1 \\ x+1, & x \geq -1 \end{cases}$ 1

$\int_{-4}^{2} |x+1|dx = \int_{-4}^{-1} (-x-1)\,dx + \int_{-1}^{2} (x+1)\,dx$

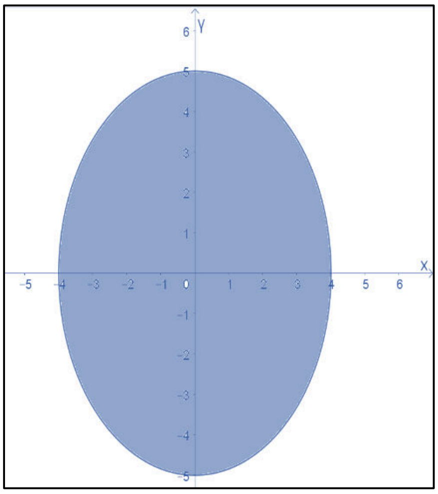
$= -\frac{(x+1)^2}{2}\Big]_{-4}^{-1} + \frac{(x+1)^2}{2}\Big]_{-1}^{2}$ 1

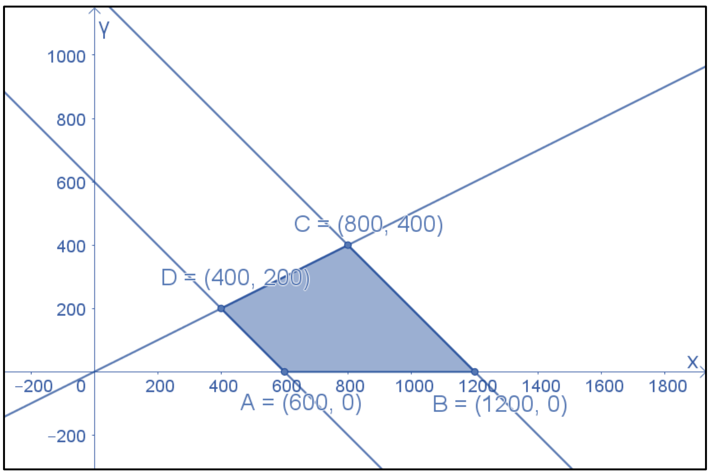
$= -\left(0 - \frac{9}{2}\right) + \left(\frac{9}{2} - 0\right) = 9$

It represent the area of shaded region bounded by the curve $y = |x+1|$, 1

$x -$ axis and the lines $x = -4$ *and* $x = 2$

OR



	$25x^2+16y^2=400 \Rightarrow \frac{x^2}{16}+\frac{y^2}{25}=1 \Rightarrow \frac{x^2}{4^2}+\frac{y^2}{5^2}=1 \Rightarrow y=\frac{5}{4}\sqrt{4^2-x^2}$ Required Area = $4\int_0^4 \frac{5}{4}\sqrt{4^2-x^2}\,dx$ $=5\left[\frac{x\sqrt{4^2-x^2}}{2}+\frac{4^2}{2}\sin^{-1}\left(\frac{x}{4}\right)\right]_0^4$ $=5[0+8\sin^{-1}(1)-0]$ $=40\times\frac{\pi}{2}=20\pi$ sq. units	1 1 1
29A 29B	The line through $(2,-1,3)$ parallel to the z-axis is given by $\vec{r}=(2\hat{\imath}-\hat{\jmath}+3\hat{k})+\lambda(\hat{k})$ Any point on this line is P $(2,-1,3+\lambda)$ Any point on the given line $\vec{r}=(2\hat{\imath}-\hat{\jmath}+2\hat{k})+\mu(3\hat{\imath}+6\hat{\jmath}+2\hat{k})$ is Q $(2+3\mu,-1+6\mu,2+2\mu)$ For the intersection point Q $(2+3\mu,-1+6\mu,2+2\mu)$ = P (2, -1, 3+ λ) $\Rightarrow 2=2+3\mu \Rightarrow \mu=0$ The point of intersection is $(2,-1,2)$ The distance from $(2,-1,3)$ to $(2,-1,2)$ is clearly 1 unit. **Alternative Solution:** Any point on the line through $(2,-1,3)$ parallel to the z-axis is $(2,-1,\lambda)$ Any point on the given line is $(2+3\mu,-1+6\mu,2+2\mu)$ Therefore, $2=2+3\mu \Rightarrow \mu=0$ The point of intersection is $(2,-1,2)$ The distance from $(2,-1,3)$ to $(2,-1,2)$ is clearly 1 unit. **OR** The line through $(2,-1,1)$parallel to the z-axis is $\vec{r}=(2\hat{\imath}-\hat{\jmath}+\hat{k})+\lambda(\hat{k})$ Any point on this line is P (2, −1, 1+ λ) Any point on the given line is A (3+ μ, μ, 1+ μ) A (3+ μ, μ, 1+ μ) = P (2, −1, 1+ λ) $\Rightarrow \mu=-1$ The point of intersection is (2, −1,0) The distance of $(2,-1,0)$ from the z-axis is $\sqrt{2^2+(-1)^2}=\sqrt{5}$ units.	1 ½ ½ ½ ½ 1 1 ½ ½ 1 1 ½ ½
30	Sketching the graph  	$1\frac{1}{2}$

30	Corner points A(600,0), B(1200,0), C(800,400), D(400,200) Values of Z: $Z_A = 1200, Z_B = 2400, Z_C = 2000, Z_D = 1000$ Maximum $Z = 2400$ when $x = 1200$ and $y = 0$ **For Visually Impaired:** At Corner points A(600,0), B(1200,0), C(800,400), D(400,200) Values of Z are $Z_A = 1800, Z_B = 3600, Z_C = 3200, Z_D = 1600$ Maximum Value of Z = 3600 at B(1200,0) And Minimum Value of Z= 1600 at D(400,200)	½ ½ ½ 1 1 1
31	Let the events be: A: Mehul is selected B: Rashi is selected Then according to the question, A and B are independent events and $P(A) = 0.4, \quad P(A \cap \bar{B}) + P(B \cap \bar{A}) = 0.5$ Let $P(B) = x$ Then $P(A \cap \bar{B}) + P(B \cap \bar{A}) = 0.5$ $\Rightarrow P(A)P(\bar{B}) + P(B)P(\bar{A}) = 0.5$ $\Rightarrow 0.4(1 - x) + x(1 - 0.4) = 0.5$ $\Rightarrow 0.4 - 0.4x + 0.6x = 0.5$ $\Rightarrow 0.2x = 0.5 - 0.4 = 0.1$ $\Rightarrow x = \frac{0.1}{0.2} = \frac{1}{2} = 0.5$ So, probability of selection of Rashi = 0.5 Probability of selection of at least one of them $= 1 - P(\bar{A} \cap \bar{B})$ $= 1 - P(\bar{A})P(\bar{B})$ $= 1 - 0.6 \times 0.5$ $= 1 - 0.3 = 0.7$	1 1 1
	SECTION D **(LA type questions of 5 marks each)**	
32	$AB = \begin{bmatrix} 3 & -6 & -1 \\ 2 & -5 & -1 \\ -2 & 4 & 1 \end{bmatrix} \begin{bmatrix} 1 & -2 & -1 \\ 0 & -1 & -1 \\ 2 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$ So, $A^{-1} = B \text{ and } B^{-1} = A$ Given system of equations is $3x - 6y - z = 3, \; 2x - 5y - z + 2 = 0, \; -2x + 4y + z = 5$ In matrix form it can be written as: $AX = C$, where $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $C = \begin{bmatrix} 3 \\ -2 \\ 5 \end{bmatrix}$ Here $\|A\| = -3 - 0 + 2 = -1 \neq 0$ So, the system is consistent and has unique solution given by the expression $X = A^{-1}C = BC$ $\Rightarrow X = \begin{bmatrix} 1 & -2 & -1 \\ 0 & -1 & -1 \\ 2 & 0 & 3 \end{bmatrix} \begin{bmatrix} 3 \\ -2 \\ 5 \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \\ 21 \end{bmatrix}$ Thus $x = 2, y = -3, z = 21$	1 ½ ½ ½ ½ ½ 1 ½

33A	Let $x = \tan\theta \Rightarrow dx = sec^2\theta\ d\theta$ $I = \int_0^{\frac{\pi}{4}} \frac{\log(1+tan\theta)}{1+tan^2\theta} . sec^2\theta\ \ d\theta$ $I = \int_0^{\frac{\pi}{4}} log\ (1 + tan\theta)d\theta \quad = \quad \int_0^{\frac{\pi}{4}} log\ \left[1 + \tan\left(\frac{\pi}{4} - \theta\right)\right] d\theta$ $= \int_0^{\frac{\pi}{4}} \log\left[1 + \frac{1-\tan\theta}{1+\tan\theta}\right]\ d\theta$ $= \int_0^{\frac{\pi}{4}} \log\left[\frac{1+\tan\theta+1-\tan\theta}{1+\tan\theta}\right] d\theta$ $= \int_0^{\frac{\pi}{4}} \log\left[\frac{2}{1+\tan\theta}\right]\ d\theta$ $= \int_0^{\frac{\pi}{4}} \log 2\ \ d\theta\ \ - \int_0^{\frac{\pi}{4}} \log[1 + tan\theta]\ \ d\theta$ $= \log 2 \times x]_0^{\frac{\pi}{4}} - I$ $\Rightarrow\ \ 2I = \frac{\pi}{4}\log 2$ $\Rightarrow\ \ I = \frac{\pi}{8}\log 2$	½ 1 1 1 1 ½
33B	**OR** I = $\int \frac{(3\ \sin\theta-2)\cos\theta}{5-cos^2\theta-4\sin\theta} d\theta = \int \frac{(3\ sin\,\theta-2)\ cos\,\theta}{5-(1-sin^2\theta)-4\,sin\,\theta}\ d\theta$ $Let\quad sin\,\theta = t\ \Longrightarrow cos\,\theta\ \ d\theta = dt$ $I = \int \frac{(3t-2)}{5-(1-t^2)-4t}\ dt$ $= \int \frac{(3t-2)}{t^2-4t+4}\ dt = \int \frac{3t-2}{(t-2)^2}\ dt$ Let $\frac{3t-2}{(t-2)^2} = \frac{A}{(t-2)} + \frac{B}{(t-2)^2}$ $3t - 2 = A(t-2) + B$ Comparing the coefficients of t and constant terms on both sides $A\ =\ 3\ , -2A + B\ =\ -2\ , B\ =\ 4$ $\int \frac{(3\ \sin\theta-2)\cos\theta}{5-cos^2\theta-4\sin\theta}\ d\theta = \int \frac{3}{t-2}\ dt + \int \frac{4}{(t-2)^2}$ dt $= 3\log\|t-2\|\ \ -\ \ \frac{4}{t-2} + C$ $= 3\ \log\|sin\,\theta - 2\|\ - \frac{4}{\sin\theta-2} + C$	OR ½ 1 ½ +½ 1+1 ½
34A	$y + \frac{d}{dx}(\ xy\ ) = x\ (\sin x + x)$ $\Longrightarrow y + (x\frac{dy}{dx} + y\ ) = x\ (\sin x + x)$ $\Longrightarrow 2y + x\frac{dy}{dx} = x\ (\sin x + x)$ $\Longrightarrow \frac{dy}{dx} + \frac{2y}{x} = (\sin x + x)$ This a linear differential equation of the form $\frac{dy}{dx} + Py = Q$ P= $\frac{2}{x}$,Q= $(\sin x + x)$ I.F $= e^{\int \frac{2}{x}\ dx} = \ e^{2\ log\ x} = e^{log\ x^2} = x^2$ Solution will be y . I.F $= \int Q.IF\ dx$ $yx^2 = \int(sin\ x + x)\ x^2$ dx $yx^2 = \int sinx.x^2\ dx + \int x^3\ dx$	 1 1 1

	$\Rightarrow \quad yx^2 = -x^2 \cos x + 2\int x \cos x \, dx \ + \frac{x^4}{4}$ + C $\Rightarrow yx^2 = -x^2 cos\ x$ +2 $(x\ sinx + cos\ x)$ + $\frac{x^4}{4}$ + C Which is the required solution	1 1
	OR	
34B	$2y\, e^{x/y} dx + (\, y - 2x\, e^{x/y}\,)\, dy$ = 0 $\Rightarrow \quad \frac{dx}{dy} = \frac{2x\, e^{x/y} - y}{2y\, e^{x/y}} = \frac{2\frac{x}{y} e^{\frac{x}{y}} - 1}{2\, e^{\frac{x}{y}}}$ It is a homogeneous differential equation. Let $x = vy \Rightarrow \frac{dx}{dy} = v + y\frac{dv}{dy}$ $v + y\frac{dv}{dy} = \frac{2ve^v - 1}{2e^v}$ $\Rightarrow y\frac{dv}{dy} = \frac{2ve^v - 1}{2e^v} - v = \frac{2ve^v - 1 - 2ve^v}{2e^v}$ $\Rightarrow \quad y\frac{dv}{dy} = \frac{-1}{2e^v}$ $\Rightarrow 2e^v\, dv = -\frac{dy}{y}$ $\int 2e^v\, dv = -\int \frac{dy}{y}$ $\Rightarrow 2e^v = -\log\|y\|$ + C $\Rightarrow 2e^{\frac{x}{y}}$ +" $log\|y\|$ = C When $x = 0$, $y = 1$, C = 2 Required solution 2 $e^{\frac{x}{y}}$ + $\log\|y\|$ = 2	1 1 1 1 1
35	Let $\frac{x-1}{3} = \frac{y-0}{-1} = \frac{z+1}{0} = \lambda \quad \Rightarrow$ Any point on it is $(3\,\lambda + 1, -\lambda, -1)$ For the point where $y = 1 \quad \Rightarrow \quad \lambda = -1$ $\Rightarrow$ The point is $(-2, 1\,, -1)$ The directions of the two lines are $\vec{m} = 3\hat{\imath} - \hat{\jmath}$ and $\vec{n} = -2\hat{\imath} + 2\hat{\jmath} + \hat{k}$ $\vec{m} \times \vec{n} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 3 & -1 & 0 \\ -2 & 2 & 1 \end{vmatrix} = -\hat{\imath} - 3\hat{\jmath} + 4\hat{k}$ The equation of the required line is $\vec{r} = \left(-2\hat{\imath} + \hat{\jmath} - \hat{k}\right) + \mu\left(-\hat{\imath} - 3\hat{\jmath} + 4\hat{k}\right)$ **Alternative Solution:** Let $\frac{x-1}{3} = \frac{y-0}{-1} = \frac{z+1}{0} = \lambda \ \Rightarrow$ Any point on it is $(3\,\lambda + 1, -\lambda, -1)$ For the point where $y = 1 \quad \Rightarrow \quad \lambda = -1$ $\Rightarrow$ The point is $(-2, 1, -1)$ Let the direction ratios of the required line be a, b, c Then $3a - b = 0$ And $-2a + 2b + c = 0$ Solving we get $\frac{a}{-1} = \frac{-b}{3} = \frac{c}{4} \Rightarrow \frac{a}{-1} = \frac{b}{-3} = \frac{c}{4}$ The required line is $\frac{x+2}{-1} = \frac{y-1}{-3} = \frac{z+1}{4} = \mu$ In vector form $\vec{r} = \left(-2\hat{\imath} + \hat{\jmath} - \hat{k}\right) + \mu\left(-\hat{\imath} - 3\hat{\jmath} + 4\hat{k}\right)$	½ 1 ½ 1 ½ 1 ½ ½ 1 ½ 1 1 ½ ½

SECTION- E (3 case-study/passage-based questions of 4 marks each)		
36	I. Traffic flow is not reflexive as $(A,A) \notin R$ (or no major spot is connected with itself) II. Traffic flow is not transitive as $(A,B) \in R\ and\ (B,E) \in R, but\ (A,E) \notin R$ III A. $R = \{(A,B),(A,C),(A,D),(B,C),(B,E),(C,E),(D,E),(D,C)\}$ Domain $= \{A,B,C,D\}$ Range $= \{B,C,D,E\}$ **OR** III B. No, the traffic flow doesn't represent a function as A has three images.	1 1 1 ½ + ½ 1+1
37	I. $P(x) = R(x) - C(x) = -0.3x^2 + 20x - (0.5x^2 - 10x + 150)$ $= -0.8x^2 + 30x - 150$ II. For critical points $P'(x) = 0 \Rightarrow -1.6x + 30 = 0$ $\Rightarrow x = \frac{30}{1.6} = \frac{300}{16} = 18.75$ III A. Now $P''(x) = -1.6$ In particular $P''(18.75) = -1.6 < 0$ So, critical value $x = 18.75$ corresponds to a maximum profit. **OR** III B. As x is the number of bulbs, so practically 18 bulbs correspond to a maximum profit. Maximum profit is $P(18) = -0.8 \times 18^2 + 30 \times 18 - 150$ $= -259.2 + 540 - 150$ $= 540 - 409.2 = ₹130.80$	1 1 1 1 1 1
38	Let the events be E_1: the student is in the first group (time spent on screen is more than 4 hours) E_2: the student is in the second group (time spent on screen is 2 to 4 hours) E_3: the student is in third group (time spent on screen is less than 2 hours) A: the event of the student showing symptoms of anxiety and low retention $P(E_1) = \frac{60}{100}$ $P(E_2) = \frac{30}{100}$ and $P(E_3) = \frac{10}{100}$ $P(A\|E_1) = \frac{80}{100}$ $P(A\|E_2) = \frac{70}{100}$ and $P(A\|E_3) = \frac{30}{100}$ I. $P(A) = P(E_1) \times P(A\|E_1) + P(E_2) \times P(A\|E_2) + P(E_3) \times P(A\|E_3)$ $= \frac{60}{100} \times \frac{80}{100} + \frac{30}{100} \times \frac{70}{100} + \frac{10}{100} \times \frac{30}{100} = \frac{72}{100} = 72\%$ II. $P(E_1\|A) = \frac{P(E_1 \cap A)}{P(A)}$ $= \frac{\left(\frac{60}{100} \times \frac{80}{100}\right)}{\left(\frac{72}{100}\right)} = \frac{48}{72} = \frac{2}{3}$	 2 2