Marking Scheme

Mathematics (Term-I)

Class-XII (Code-041)

Q.N.	Correct Option	Hints / Solutions
1	d	$sin\left(\frac{\pi}{3} - \left(\frac{-\pi}{6}\right)\right) = \sin\left(\frac{\pi}{2}\right) = 1$
2	b	$\lim\limits_{x \to o}\left(\frac{1 - \cos kx}{x \sin x}\right) = \frac{1}{2}$ $\Rightarrow \lim\limits_{x \to o}\left(\frac{2\sin^2\frac{kx}{2}}{x \sin x}\right) = \frac{1}{2}$ $\Rightarrow \lim\limits_{x \to o} 2\left(\frac{k}{2}\right)^2\left(\frac{\sin\frac{kx}{2}}{\frac{kx}{2}}\right)^2\left(\frac{x}{\sin x}\right) = \frac{1}{2}$ $\Rightarrow k^2 = 1 \Rightarrow k = \pm 1$ but $k < 0 \Rightarrow$ k= -1
3	d	$A^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
4	c	As A is singular matrix $\Rightarrow |A| = 0$ $\Rightarrow 2k^2 - 32 = 0 \Rightarrow$ k = ± 4
5	b	$f(x) = x^2 - 4x + 6$ $f'(x) = 2x - 4$ $let\ f'(x) = 0 \Rightarrow$ x = 2 $\xleftarrow{\quad}\underset{-\infty\ \ (-)\quad 2\quad (+)\quad \infty}{\xrightarrow{\quad\quad\quad}}$ as $f'(x) > 0 \ \forall \ \ x \in (2, \infty)$ $\Rightarrow f(x)$ is Strictly increasing in $(2, \infty)$
6	d	as $|adj\ A| = |A|^{n-1}$, where n is order of matrix A $\quad\quad = (-4)^2 = 16$
7	b	$(1, 2)$
8	a	$\left.\begin{array}{l} 2a + b = 4 \\ a - 2b = -3 \\ 5c - d = 11 \\ 4c + 3d = 24 \end{array}\right\} \Rightarrow \begin{array}{l} a = 1 \\ b = 2 \\ c = 3 \\ d = 4 \end{array}$ $\therefore a + b - c + 2d = 8$
9	a	$f(x) = x + \frac{1}{x}, x > 0 \Rightarrow f'(x) = 1 - \frac{1}{x^2} = \frac{x^2 - 1}{x^2}, x > 0$ As normal to $f(x)$ is $\perp$ to given line $\Rightarrow \left(\frac{x^2}{1 - x^2}\right) \times \frac{3}{4} = -1 \ (m_1 . m_2 = -1)$ $\Rightarrow x^2 = 4 \Rightarrow$ x = ± 2 But $x > 0, \therefore x = 2$ Therefore point$= \left(2, \frac{5}{2}\right)$
10	d	$\sin(\tan^{-1} x) = \sin\left(\sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)\right) = \frac{x}{\sqrt{1+x^2}}$
11	a	$\{1, 5, 9\}$
12	c	$e^x + e^y = e^{x+y}$ $\Rightarrow e^{-y} + e^{-x} = 1$ Differentiating $w.r.t.\ x$:

		$\Rightarrow -e^{-y}\frac{dy}{dx} - e^{-x} = 0 \Rightarrow \frac{dy}{dx} = -e^{y-x}$
13	b	3×5
14	a	$y = 5\cos x - 3\sin x \Rightarrow \frac{dy}{dx} = -5\sin x - 3\cos x$ $\Rightarrow \frac{d^2y}{dx^2} = -5\cos x + 3\sin x = -y$
15	c	$\text{adj } A = \begin{bmatrix} 7 & -5 \\ 11 & 2 \end{bmatrix} \Rightarrow (adjA)' = \begin{bmatrix} 7 & 11 \\ -5 & 2 \end{bmatrix}$
16	c	$\frac{x^2}{9} + \frac{y^2}{16} = 1 \Rightarrow \frac{2x}{9} + \frac{2y}{16}\frac{dy}{dx} = 0$ $\Rightarrow slope\ of\ normal = \frac{-dx}{dy} = \frac{9y}{16x}$ As curve's tangent is parallel to *y*-axes $\Rightarrow \frac{9y}{16x} = 0 \Rightarrow y = 0$ and $x = \pm 3$ $\therefore\ \ points = (\pm 3, 0)$
17	b	$\lvert A \rvert = -7$ $\therefore \sum_{i=1}^{3} a_{i2}A_{i2} = a_{12}A_{12} + a_{22}A_{22} + a_{32}A_{32} = \lvert A \rvert = -7$
18	d	$y = \log(\cos e^x)$ Differentiating wrt x: $\frac{dy}{dx} = \frac{1}{\cos(e^x)} \cdot (-\sin e^x) \cdot e^x$ (chain rule) $\Rightarrow \frac{dy}{dx} = -e^x \tan e^x$
19	d	Z is maximum 180 at points *C (15,15)* and *D(0, 20)*. $\Rightarrow$ Z is maximum at every point on the line segment *CD*
20	c	$f(x) = 2\cos x + x\ ,\ \ x \in [0, \frac{\pi}{2}]$ $f'(x) = -2\sin x + 1$ Let$f'(x) = 0 \Rightarrow x = \frac{\pi}{6}\ \ \in [0, \frac{\pi}{2}]$ $f(0) = 2$ $f\left(\frac{\pi}{6}\right) = \frac{\pi}{6} + \sqrt{3}$ $f\left(\frac{\pi}{2}\right) = \frac{\pi}{2}\ \Rightarrow$ least value of $f(x)$ is $\frac{\pi}{2}$ at $x = \frac{\pi}{2}$
		Section-B
21	d	$let\ f(x_1) = f(x_2)\ \forall x_1 x_2 \in R$ $\Rightarrow x_1^3 = x_2^3$ $\Rightarrow x_1 = x_2$ $\Rightarrow f$ is one - one $let\ f(x) = x^3 = y\ \ \forall\ y \in R$ $\Rightarrow x = y^{\frac{1}{3}}$ every image $y \in R$ has a unique pre image in R $\Rightarrow f$ is onto $\therefore f$ is one-one and onto
22	a	$x = a\sec\theta \Rightarrow \frac{dx}{d\theta} = a\tan\theta \sec\theta$ $y = b\tan\theta\ \Rightarrow \frac{dy}{d\theta} = b\sec^2\theta$ $\therefore \frac{dy}{dx} = \frac{b}{a}\cosec\theta$ $\Rightarrow \frac{d^2y}{dx^2} = \frac{-b}{a}\cosec\theta.\cot\theta.\frac{d\theta}{dx} = \frac{-b}{a^2}\cot^3\theta$ $\therefore \frac{d^2y}{dx^2}\Big]_{\theta = \frac{\pi}{6}} = \frac{-3\sqrt{3}b}{a^2}$
23	c	Z is minimum -24 at (0, 8)
24	a	$let\ u = sin^{-1}(2x\sqrt{1-x^2}\ )$

		and $v = sin^{-1}x$, $\frac{1}{\sqrt{2}} < x < 1 \Rightarrow \sin v = x$......(1) Using (1), we get : $= sin^{-1}(2 \sin v \cos v)$ $\Rightarrow u = 2v$ Differentiating with respect to v, we get: $\frac{du}{dv} = 2$
25	d	$AB = 6I \Rightarrow B^{-1} = \frac{1}{6}A$
26	b	$f'(x) = 6(x^2 - x - 6) = 6(x-3)(x+2)$ $\xleftarrow{\quad}$ $-\infty \;_{(+)} \quad -2 \quad (-) \quad 3 \quad (+) \quad \infty$ As $f'(x) < 0 \; \forall \; x \in (-2, 3)$ $\Rightarrow f(x)$ is strictly decreasing in $(-2, 3)$
27	a	$tan^{-1}\left(\frac{\sqrt{1+\cos x} + \sqrt{1-\cos x}}{\sqrt{1+\cos x} - \sqrt{1-\cos x}}\right)$ $= tan^{-1}\left(\frac{-\sqrt{2}\cos\frac{x}{2} + \sqrt{2}\sin\frac{x}{2}}{-\sqrt{2}\cos - \sqrt{2}\sin\frac{x}{2}}\right)$, $\pi < x < \frac{3\pi}{2}$ $= tan^{-1}\left(\frac{\cos\frac{x}{2} - \sin\frac{x}{2}}{\cos\frac{x}{2} + \sin\frac{x}{2}}\right)$ $= \frac{\pi}{4} - \frac{x}{2}$
28	c	$A^2 = 2A$ $\Rightarrow |A^2| = |2A|$ $\Rightarrow |A|^2 = 2^3|A|$ as $|kA| = k^n|A|$ for a matrix of order n $\Rightarrow$ either $|A| = 0$ or $|A| = 8$ But A is non-singular matrix $\therefore |A| = 8^2 = 64$
29	b	$f'(x) = 1 - \sin x \Rightarrow f'(x) > 0 \; \forall x \in R$ $\Rightarrow$ no value of b exists
30	c	$a = b - 2$ and $b > 6$ $\Rightarrow (6, 8) \in R$
31	a	$f(x) = \begin{cases} \frac{x}{-x} = -1 , & x < 0 \\ -1 , & x \geq 0 \end{cases}$ $\Rightarrow f(x) = -1 \; \forall \; x \in R$ $\Rightarrow f(x)$ is continous $\forall \; x \in R$ as it is a constant function
32	b	$kA = \begin{bmatrix} 0 & 2k \\ 3k & -4k \end{bmatrix} = \begin{bmatrix} 0 & 3a \\ 2b & 24 \end{bmatrix}$ $\Rightarrow k = -6, a = -4$ and $b = -9$
33	d	Corner points of feasible region $\quad Z = 30x + 50y$ (5,0) $\qquad\qquad\qquad\quad$ 150 (9,0) $\qquad\qquad\qquad\quad$ 270 (0,3) $\qquad\qquad\qquad\quad$ 150 (0,6) $\qquad\qquad\qquad\quad$ 300 Minimum value of Z occurs at two points
34	c	$f'(x) = \frac{-2x^2 - 10x + 100}{\sqrt{100 - x^2}}$ $f'(x) = 0 \Rightarrow x = -10$ or 5 , But $x > 0 \Rightarrow x = 5$ $f''(x) = \frac{2x^3 - 300x - 1000}{(100-x)^{\frac{3}{2}}} \Rightarrow f''(5) = \frac{-30}{\sqrt{75}} < 0$ $\Rightarrow$ Maximum area of trapezium is $75\sqrt{3} \; cm^2$ when $x = 5$
35	d	$(I - A)^3 - 7A = I + A + 3A + 3A - 7A = I$
36	c	$\frac{-\pi}{2} < y < \frac{\pi}{2}$

37	b	As every per-image $x \in A$ has a unique image $y \in B$ $\Rightarrow f$ is injective function
38	b	$\|A\| = 7$, $adjA = \begin{bmatrix} 2 & -2 \\ 1 & 3 \end{bmatrix}$ $\therefore 14A^{-1} = \begin{bmatrix} 4 & -2 \\ 2 & 6 \end{bmatrix}$
39	b	$y = x^3 - 11x + 5 \Rightarrow \frac{dy}{dx} = 3x^2 - 11$ Slope of line $y = x - 11$ is $1 \Rightarrow 3x^2 - 11 = 1 \Rightarrow x = \pm 2$ $\therefore$ point is (2, -9) as (-2, 19) does not satisfy given line
40	c	$A^2 = 3I$ $\Rightarrow \begin{bmatrix} \alpha^2 + \beta r & 0 \\ 0 & \beta r + \alpha^2 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} \Rightarrow 3 - \alpha^2 - \beta r = 0$

Section C

41	a	As Z is maximum at (30, 30) and (0, 40) $\Rightarrow 30a + 30b = 40b \Rightarrow b - 3a = 0$
42	b	$y = mx + 1 \dots(1)$ and $y^2 = 4x \dots(2)$ Substituting (1) in (2): $(mx + 1)^2 = 4x$ $\Rightarrow m^2 x^2 + (2m - 4)x + 1 = 0 \dots(3)$ As line is tangent to the curve $\Rightarrow$ line touches the curve at only one point $\Rightarrow (2m - 4)^2 - 4m^2 = 0 \Rightarrow m = 1$
43	c	Let $f(x) = [x(x - 1) + 1]^{\frac{1}{3}}$, $0 \leq x \leq 1$ $f'(x) = \frac{2x - 1}{3(x^2 - x + 1)^{\frac{2}{3}}}$ let $f'(x) = 0 \Rightarrow x = \frac{1}{2} \in [0, 1]$ $f(o) = 1, f\left(\frac{1}{2}\right) = \left(\frac{3}{4}\right)^{\frac{1}{3}}$ and $f(1) = 1$ $\therefore$ Maximum value of $f(x)$ is 1
44	b	Feasible region is bounded in the first quadrant
45	d	$\|A\| = 2 + 2sin^2\theta$ As $-1 \leq sin\theta \leq 1, \forall\ 0 \leq \theta \leq 2\pi$ $\Rightarrow 2 \leq 2 + 2sin^2\theta \leq 4 \Rightarrow \|A\| \in [2, 4]$
46	d	Fuel cost$= k(speed)^2$ $\Rightarrow 48 = k. 16^2 \Rightarrow k = \frac{3}{16}$
47	b	Total cost of running train (let C) $=\frac{3}{16}v^2 t + 1200t$ Distance covered $= 500km \Rightarrow$ time $= \frac{500}{v}hrs$ Total cost of running train 500 km$= \frac{3}{16}v^2 \left(\frac{500}{v}\right) + 1200\left(\frac{500}{v}\right)$ $\Rightarrow C = \frac{375}{4}v + \frac{600000}{v}$
48	c	$\frac{dC}{dv} = \frac{375}{4} - \frac{600000}{v^2}$ Let $\frac{dC}{dv} = 0 \Rightarrow$ v $= 80$ km/h
49	c	Fuel cost for running 500 km $\frac{375}{4}v = \frac{375}{4} \times 80 = Rs. 7500/-$
50	d	Total cost for running 500 km $= \frac{375}{4}v + \frac{600000}{v}$ $= \frac{375 \times 80}{4} + \frac{600000}{80} = Rs. 15000/-$