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Class: XII Session: 2020-21

Subject: Mathematics

Marking Scheme (Theory)

Sr.No.	Objective type Question Section I	Marks
1	Let $f(x_1) = f(x_2)$ for $some\ x_1, x_2\ \in R$ $\Rightarrow\ (x_1)^3 = (x_2)^3$ $\Rightarrow\ x_1 = x_2,$ Hence $f(x)$ is $one-one$ **OR** 2^6 reflexive relations	1 1
2	(1,2)	1
3	Since $\sqrt{a}$ is not defined for $a \in (-\infty, 0)$ $\therefore\ \sqrt{a} = b\ is\ not\ a\ function.$ **OR** $A_1 \cup A_2 \cup A_3 = A\ and\ A_1 \cap A_2 \cap A_3 = \phi$	1 1
4	3x5	1
5	$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \Rightarrow A^2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ OR |adj A|=(-4)$^{3\text{-}1}$=16	1
6	0	1
7	$e^x(1 - \cot x) + $ C **OR** $\because f(x)$ is an odd function $\therefore \displaystyle\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} x^2 \sin x\ dx = 0$	1 1
8	$A = 2 \displaystyle\int_{0}^{1} x^2\ dx = \frac{2}{3}[x^3]_0^1$ $= \frac{2}{3} sq\ unit$	1

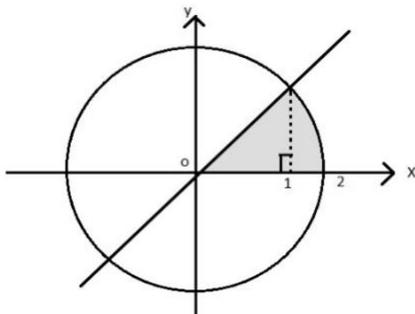| 9 | 0 | 1 |
	OR	
	3	1
10	$\hat{j}$	1
11	$\frac{1}{2}\lvert 2\hat{i} \times (-3\hat{j})\rvert = \frac{1}{2}\left\lvert -6\hat{k}\right\rvert = 3\ sq\ units$	1
12	$\begin{aligned}&\left\lvert \hat{a} + \hat{b}\right\rvert^2 = 1\\ &\Rightarrow\ \hat{a}^2 + \hat{b}^2 + 2\,\hat{a}.\hat{b} = 1\\ &\Rightarrow\ 2\,\hat{a}.\hat{b} = 1 - 1 - 1\\ &\Rightarrow\ \hat{a}.\hat{b} = \frac{-1}{2} \Rightarrow \lvert\hat{a}\rvert\lvert\hat{b}\rvert\cos\theta = \frac{-1}{2} \Rightarrow \theta = \pi - \frac{\pi}{3}\\ &\Rightarrow\ \theta = \frac{2\pi}{3}\end{aligned}$	1
13	1,0,0	1
14	(0,0,0)	1
15	$1 - \frac{2}{3} \times \frac{3}{4} = \frac{1}{2}$	1
16	$\left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right)^3 = \left(\frac{1}{2}\right)^7$	1
	Section II	
17(i)	(b)	1
17(ii)	(a)	1
17(iii)	(c)	1
17(iv)	(a)	1
17(v)	(d)	1
18(i)	(b)	1
18(ii)	(c)	1
18(iii)	(b)	1
18(iv)	(d)	1
18(v)	(d)	1
	Section III	
19	$tan^{-1}\left(\frac{cosx}{1-sinx}\right) = tan^{-1}\left[\frac{\sin\left(\frac{\pi}{2}-x\right)}{1-\cos\left(\frac{\pi}{2}-x\right)}\right]$ $tan^{-1}\left[\frac{2\sin\left(\frac{\pi}{4}-\frac{x}{2}\right)\cos\left(\frac{\pi}{4}-\frac{x}{2}\right)}{2\,sin^2\left(\frac{\pi}{4}-\frac{x}{2}\right)}\right]$	$\frac{1}{2}$

	$tan^{-1}\left[cot\left(\frac{\pi}{4}-\frac{x}{2}\right)\right] = tan^{-1}\left[tan\frac{\pi}{2}-\left(\frac{\pi}{4}-\frac{x}{2}\right)\right]$	1
	$tan^{-1}\left[tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\right] = \frac{\pi}{4}+\frac{x}{2}$	$\frac{1}{2}$
20	$A^2 = 2A$ $\Rightarrow |AA| = |2A|$ $\Rightarrow |A||A| = 8|A| \quad (\because |AB| = |A||B| \text{ and } |2A| = 2^3|A|)$ $\Rightarrow |A|(|A|-8) = 0$ $\Rightarrow |A| = 0 \text{ or } 8$	$\frac{1}{2}$ 1 $\frac{1}{2}$
	OR $A^2 = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$ $5A = \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix}, 7I = \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$ $\Rightarrow A^2 - 5A + 7I = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$ $\Rightarrow A^{-1}(A^2 - 5A + 7I) = A^{-1}0$ $\Rightarrow A - 5I + 7A^{-1} = 0$ $\Rightarrow 7A^{-1} = 5I - A$ $\Rightarrow A^{-1} = \frac{1}{7}\left(\begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\right)$ $\Rightarrow A^{-1} = \frac{1}{7}\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$	1 1
21	$\underset{x \to 0}{Lt}\ \frac{1 - \cos kx}{x \sin x} = \underset{x \to 0}{Lt}\ \frac{2 \sin^2\left(\frac{kx}{2}\right)}{x \sin x}$ $= \underset{x \to 0}{Lt}\ \frac{\frac{2 \sin^2\left(\frac{kx}{2}\right)}{x^2}}{\frac{x \sin x}{x^2}}$ $= \frac{\underset{x \to 0}{Lt}\ \frac{2 \sin^2\left(\frac{kx}{2}\right)}{\left(\frac{kx}{2}\right)^2} \times \left(\frac{k}{2}\right)^2}{\underset{x \to 0}{Lt}\ \frac{\sin x}{x}} = \frac{2 \times 1 \times \frac{k^2}{4}}{1}$	$1\frac{1}{2}$

	$\because f(x)$ is continuous at $x = 0$ $\therefore \underset{x \to 0}{Lt} f(x) = f(0)$ $\Rightarrow \dfrac{k^2}{2} = \dfrac{1}{2} \Rightarrow k^2 = 1 \Rightarrow k = \pm 1$	$\dfrac{1}{2}$
22	$y = x + \dfrac{1}{x} \Rightarrow \dfrac{dy}{dx} = 1 - \dfrac{1}{x^2}$ $\because$ normal is perpendicular to $3x - 4y = 7$, $\therefore$ tangent is parallel to it $1 - \dfrac{1}{x^2} = \dfrac{3}{4} \Rightarrow x^2 = 4 \qquad \Rightarrow x = 2 \ (\because x > 0)$ when $x = 2$, $y = 2 + \dfrac{1}{2} = \dfrac{5}{2}$ $\therefore$ Equation of Normal : $y - \dfrac{5}{2} = -\dfrac{4}{3}(x - 2) \Rightarrow \qquad 8x + 6y = 31$	1 1
23	$I = \displaystyle\int \dfrac{1}{cos^2 x \ (1 - \tan x)^2} dx$ Put, $\quad 1 - \tan x = y$ So that, $-sec^2 x \ dx = dy$ $= \displaystyle\int \dfrac{-1 \, dy}{y^2} = -\int y^{-2} dy$ $= +\dfrac{1}{y} + c = \dfrac{1}{1 - \tan x} + c$ **OR** $I = \displaystyle\int_0^1 x \ (1 - x)^n dx$ $I = \displaystyle\int_0^1 (1 - x)[1 - (1 - x)]^n dx$ $I = \displaystyle\int_0^1 (1 - x) \ x^n \ dx = \int_0^1 (x^n - x^{n+1}) dx$ $I = \left[\dfrac{x^{n+1}}{n+1} - \dfrac{x^{n+2}}{n+2} \right]_0^1$ $I = \left[\left(\dfrac{1}{n+1} - \dfrac{1}{n+2} \right) - 0 \right] = \dfrac{1}{(n+1)\,(n+2)}$	1 1 $\dfrac{1}{2}$ 1 $\dfrac{1}{2}$
24	$Area = 2 \displaystyle\int_0^2 \sqrt{8x} \ dx$ $= 2 \times 2\sqrt{2} \displaystyle\int_0^2 x^{\frac{1}{2}} \ dx$	1

	$= 4\sqrt{2}\left[\frac{2}{3}x^{\frac{3}{2}}\right]_0^2$	$\frac{1}{2}$
	$= \frac{8}{3}\sqrt{2}\left[2^{\frac{3}{2}} - 0\right] = \frac{8\sqrt{2}}{3} \times 2\sqrt{2}$	
	$= \frac{32}{3}\ sq\ units$	$\frac{1}{2}$
25	$\frac{dy}{dx} = x^3 cosec\ y\ ;\quad y(0) = 0$	
	$\int \frac{dy}{cosec\ y} = \int x^3 dx$	$\frac{1}{2}$
	$\int \sin y\ dy = \int x^3 dx$	
	$-\cos y = \frac{x^4}{4} + c$	1
	$-1 = c \quad (\because y = 0, when\ x = 0)$	
	$\cos y = 1 - \frac{x^4}{4}$	$\frac{1}{2}$
26	Let $\overrightarrow{a} = \hat{\imath} - \hat{\jmath} + \hat{k}$	
	$\overrightarrow{d} = 4\ \hat{\imath} + 5\hat{k}$	
	$\because \overrightarrow{a} + \overrightarrow{b} = \overrightarrow{d} \therefore \overrightarrow{b} = \overrightarrow{d} - \overrightarrow{a} \qquad = 3\ \hat{\imath} + \hat{\jmath} + 4\ \hat{k}$	$\frac{1}{2}$
	$\overrightarrow{a}\ x\ \overrightarrow{b} = \begin{vmatrix} \hat{\imath} & -\hat{\jmath} & +\hat{k} \\ 1 & -1 & 1 \\ 3 & 1 & 4 \end{vmatrix} = -5\hat{\imath} - 1\hat{\jmath} + 4\ \hat{k}$	1
	Area of parallelogram $= \left|\overrightarrow{a}\ x\ \overrightarrow{b}\right| = \sqrt{25 + 1 + 16} = \sqrt{42}\ sq\ units$	$\frac{1}{2}$
27	Let the normal vector to the plane be $\overrightarrow{n}$ Equation of the plane passing through (1,0,0), i.e., $\hat{\imath}$ is $(\overrightarrow{r} - \hat{\imath}) \cdot \overrightarrow{n} = 0$ ....(1)	1
	$\because$ plane (1) contains the line $\overrightarrow{r} = \overrightarrow{o} + \lambda\ \hat{\jmath}$	
	$\therefore \hat{\imath} \cdot \overrightarrow{n} = 0$ and $\hat{\jmath} \cdot \overrightarrow{n} = 0 \quad \Rightarrow \overrightarrow{n} = \hat{k}$	
	Hence equation of the plane is $(\overrightarrow{r} - \hat{\imath}) \cdot \hat{k} = 0$ i.e., $\overrightarrow{r} \cdot \hat{k} = 0$	1
28	Let x denote the number of milk chocolates drawn	
	<table><tr><td>X</td><td>P(x)</td></tr></table>	

		0	$\frac{4}{6} \times \frac{3}{5} = \frac{12}{30}$		
		1	$\left(\frac{2}{6} \times \frac{4}{5}\right) \times 2 = \frac{16}{30}$		
		2	$\frac{2}{6} \times \frac{1}{5} = \frac{2}{30}$		$1\frac{1}{2}$

Most likely outcome is getting one chocolate of each type

$\frac{1}{2}$

OR

P $(\bar{E} \mid \bar{F}) = P \frac{(\bar{E} \cap \bar{F})}{P (\bar{F})} = \frac{(\overline{E \cup F})}{P (\bar{F})} = \frac{1 - P (E \cup F)}{1 - P(F)}$ ----------(1)

1

Now $P (E \cup F)$ = P (E) + P (F) - P (E ∩ F)

= 0.8+0.7-0.6=0.9

$\frac{1}{2}$

Substituting value of $P (E \cup F)$ in (1)

P $(\bar{E} \mid \bar{F}) = \frac{1 - 0.9}{1 - 0.7} = \frac{0.1}{0.3} = \frac{1}{3}$

$\frac{1}{2}$

| **Section IV**		

29	(i) Reflexive :
	    Since, a+a=2a which is even ∴ (a,a) ∈ R $\forall a \in$ Z
	    Hence R is reflexive
		$\frac{1}{2}$

	(ii) Symmetric: If (a,b) ∈R, then a+b = 2λ ⇒ b+a = 2 λ ⇒ (b,a) ∈R, Hence R is symmetric	1

	(iii) Transitive:
	    If (a,b) ∈R and (b,c,) ∈R
	    then a+b = 2 λ---(1) and b+c =2μ ---- (2)
	    Adding (1) and (2) we get
	    a+2b+c=2(λ + μ)
	  ⇒ a+c=2 (λ + $\mu - b$)
	  ⇒ a+c=2k ,where λ + μ − b = k ⇒ (a,c) ∈R
	    Hence R is transitive
	    [0] = {...-4, -2, 0, 2, 4...}
		1
		$1\frac{1}{2}$

30	Let u = $e^{x \, sin^2 \, x}$ and v = $(\sin x)^x$	$\frac{1}{2}$

	so that y = u + v $\Rightarrow \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$ ----(1) Now, u = $e^{x\sin^2 x}$, Differentiating both sides w.r.t. x, we get $\Rightarrow \frac{du}{dx} = e^{x\sin^2 x}[x(sin2x) + sin^2 x]$ ----- (2) Also , v = $(\sin x)^x$ $\Rightarrow \log v = x \log (\sin x)$ Differentiating both sides w.r.t. x, we get $\frac{1}{v}\frac{dv}{dx} = x \cot x + \log (sinx)$ $\frac{dv}{dx} = (\sin x)^x [x\, cotx + \log(sinx)]$ ------ (3) Substituting from $-(2), -(3)$ in $-(1)$ we get $\frac{dy}{dx} = e^{x\sin^2 x}[xsin2x + sin^2 x] + (\sin x)^x [x\, cotx + \log(sinx)]$	1 1 $\frac{1}{2}$
31	RHD $= \underset{h\to 0}{Lt}\frac{f(1+h) - f(1)}{h} = \underset{h\to 0}{Lt}\frac{[1+h] - [1]}{h}$ $= \underset{h\to 0}{Lt}\frac{(1-1)}{h} = 0$ LHD $= \underset{h\to 0}{Lt}\frac{f(1-h) - f(1)}{-h} = \underset{h\to 0}{Lt}\frac{[1-h] - [1]}{-h} = \underset{h\to 0}{Lt}\frac{0-1}{-h}$ $= \underset{h\to 0}{Lt}\frac{1}{h} = \infty$ Since, RHD ≠LHD Therefore f(x) is not differentiable at x = 1 **OR** $y = b\tan\theta \Rightarrow \frac{dy}{d\theta} = b\,sec^2\theta \dots (1)$ $x = a\sec\theta \Rightarrow \frac{dx}{d\theta} = a\sec\theta\tan\theta \dots (2)$	1 1 1

	$\dfrac{dy}{dx} = \dfrac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \dfrac{b\,sec^2\theta}{a\,\sec\theta\tan\theta} = \dfrac{b}{a}\,cosec\,\theta$	$1\frac{1}{2}$
	Differentiating both sides w.r.t.x, we get	
	$\dfrac{d^2y}{dx^2} = \dfrac{-b}{a}\,cosec\,\theta\cot\theta \times \dfrac{d\theta}{dx}$	
	$\qquad = \dfrac{-b}{a}\,cosec\,\theta\cot\theta \times \dfrac{1}{a\sec\theta\tan\theta}$ $[using\ (2)]$	
	$\qquad = \dfrac{-b}{a.a}\,cot^3\theta$	1
	$\dfrac{d^2y}{dx^2}\Big]_{\theta=\frac{\pi}{6}} = \dfrac{-b}{a}\Big[\cot\dfrac{\pi}{6}\Big]^3 = \dfrac{-b}{a}\big(\sqrt{3}\big)^3 = -\dfrac{3\sqrt{3}b}{a.a}$	$\frac{1}{2}$
32	$\qquad\qquad\qquad f\,(x) = \tan x - 4x$	
	$\qquad\qquad\qquad f'(x) = sec^2 x - 4$	$\frac{1}{2}$
	a) For $f\,(x)$ to be strictly increasing	
	$\qquad f'(x) > 0$	
	$\qquad \Rightarrow \qquad sec^2 x - 4 > 0$	
	$\qquad \Rightarrow \qquad sec^2 x > 4$	
	$\qquad \Rightarrow \qquad cos^2 x < \dfrac{1}{4} \Rightarrow cos^2 x < \left(\dfrac{1}{2}\right)^2$	
	$\qquad \Rightarrow \qquad -\dfrac{1}{2} < \cos x < \dfrac{1}{2} \Rightarrow \dfrac{\pi}{3} < x < \dfrac{\pi}{2}$	$1\frac{1}{2}$
	b) For $f\,(x)$ to be strictly decreasing	
	$\qquad f'(x) < 0$	
	$\qquad \Rightarrow \quad sec^2 x - 4 < 0$	
	$\qquad \Rightarrow \quad sec^2 x < 4$	
	$\qquad \Rightarrow \quad cos^2 x > \dfrac{1}{4}$	
	$\qquad \Rightarrow \quad cos^2 x > \left(\dfrac{1}{2}\right)^2$	
	$\qquad \Rightarrow \quad \cos x > \dfrac{1}{2} \ \Big[\because x \in \left(0, \dfrac{\pi}{2}\right)\Big]$	
	$\qquad \Rightarrow \quad 0 < x < \dfrac{\pi}{3}$	
		1

33	Put $x^2 = y$ to make partial fractions	$\frac{1}{2}$
	$\frac{x^2 + 1}{(x^2 + 2)(x^2 + 3)} = \frac{y + 1}{(y + 2)(y + 3)} = \frac{A}{y + 2} + \frac{B}{y + 3}$	
	$\Rightarrow\ y + 1 = A(y + 3) + B(y + 2)\ldots\ldots\ldots\ldots(1)$	$\frac{1}{2}$
	Comparing coefficients of y and constant terms on both sides of (1) we get	
	$\quad$ A+B = 1 and $\quad$ 3A + 2B = 1	
	$\quad$ Solving, we get A $= -1$, B $= 2$	1
	$\int \frac{x^2+1}{(x^2+2)(x^2+3)} dx = \int \frac{-1}{x^2+2} dx + 2 \int \frac{1}{x^2+3} dx$	
	$= -\frac{1}{\sqrt{2}} tan^{-1}\left(\frac{x}{\sqrt{2}}\right) + \frac{2}{\sqrt{3}} tan^{-1}\left(\frac{x}{\sqrt{3}}\right) + C$	1
34	Solving $y = \sqrt{3}x$ and $x^2 + y^2 = 4$	
	We get $x^2 + 3x^2 = 4$	
	$\Rightarrow x^2 = 1\ \Rightarrow x = 1$	$\frac{1}{2}$
		$\frac{1}{2}$
	Required Area	
	$= \sqrt{3} \int_0^1 x\ dx + \int_1^2 \sqrt{2^2 - x^2}\ dx$	$\frac{1}{2}$
	$= \frac{\sqrt{3}}{2} [x^2]_0^1 + \left[\frac{x}{2}\sqrt{2^2 - x^2} + 2 \sin^{-1}\left(\frac{x}{2}\right)\right]_1^2$	1
	$= \frac{\sqrt{3}}{2} + \left[2 \times \frac{\pi}{2} - \frac{\sqrt{3}}{2} - 2 \times \frac{\pi}{6}\right]$	
	$\frac{2\pi}{3}$ sq units	
		$\frac{1}{2}$
	OR	

	Required Area $= \frac{4}{3}\int_0^6 \sqrt{6^2 - x^2}\, dx$		$\frac{1}{2}$
			$\frac{1}{2}$
	$= \frac{4}{3}\left[\frac{x}{2}\sqrt{6^2 - x^2} + 18\, sin^{-1}\left(\frac{x}{6}\right)\right]_0^6$		1
	$= \frac{4}{3}\left[18 \times \frac{\pi}{2} - 0\right] = 12\pi\ sq\ units$		1
35	The given differential equation can be written as		
	$\frac{dy}{dx} = \frac{y + 2x^2}{x}$ $\Rightarrow$ $\frac{dy}{dx} - \frac{1}{x}y = 2x$		
	$Here\quad P = -\frac{1}{x},\ Q = 2x$		$\frac{1}{2}$
	IF $= e^{\int P dx} = e^{-\int \frac{1}{x}dx} = e^{-\log x} = \frac{1}{x}$		1
	The solutions is :		
	$y \times \frac{1}{x} = \int\left(2x \times \frac{1}{x}\right)dx$		1
	$\Rightarrow \frac{y}{x} = 2x + c$		
	$\Rightarrow y = 2x^2 + cx$		$\frac{1}{2}$
36	$|A| = 1(-1-2) - 2(-2-0) = -3 + 4 = 1$		$\frac{1}{2}$
	A is nonsingular, therefore A^{-1}exists		
	$Adj\ A = \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$		
	$\Rightarrow A^{-1} = \frac{1}{|A|}(Adj\ A) = \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$		$1\frac{1}{2}$
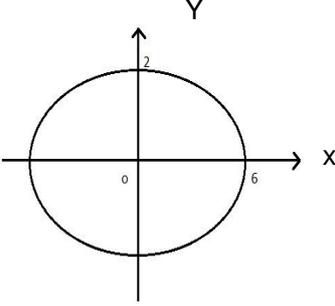
The given equations can be written as:

$$\begin{bmatrix} 1 & -2 & 0 \\ 2 & -1 & -1 \\ 0 & -2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}$$

$\frac{1}{2}$

Which is of the form $A'X = B$

$\Rightarrow X = (A')^{-1}B = (A^{-1})'B$

1

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} -3 & 2 & 2 \\ -2 & 1 & 1 \\ -4 & 2 & 3 \end{bmatrix}\begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}=\begin{bmatrix} 0 \\ -5 \\ -3 \end{bmatrix}$$

$\Rightarrow x = 0, \quad y = -5, \quad z = -3$

$1\frac{1}{2}$

OR

$$AB = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

$1\frac{1}{2}$

$\Rightarrow AB = 6I$

$\Rightarrow A\left(\dfrac{1}{6}B\right) = I \Rightarrow A^{-1} = \dfrac{1}{6}(B)$

1

The given equations can be written as

$$\begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$$

$AX = D, \quad where \ D = \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$

$\Rightarrow X = A^{-1}D$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix}=\frac{1}{6}\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}\begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}=\frac{1}{6}\begin{bmatrix} 12 \\ -6 \\ 24 \end{bmatrix}$$

1

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$$

$x = 2, \quad y = -1, \quad z = 4$

$1\frac{1}{2}$

| 37 | We have $a_1 = 3\hat{\imath} + 2\hat{\jmath} - 4\hat{k}$ $b_1 = \hat{\imath} + 2\hat{\jmath} + 2\hat{k}$ | |

$$a_2 = 5i - 2j \qquad\qquad b_2 = 3\hat{\imath} + 2\hat{\jmath} + 6\hat{k}$$

$$\vec{a_2} - \vec{a_1} = 2\hat{\imath} - 4\hat{\jmath} + 4\hat{k}$$

$$\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 2 & 2 \\ 3 & 2 & 6 \end{vmatrix} = \hat{\imath}(12-4) - \hat{\jmath}(6-6) + \hat{k}(2-6)$$

$$\vec{b1} \times \vec{b_2} = 8\hat{\imath} + 0\hat{\jmath} - 4\hat{k} = 8\hat{\imath} - 4\hat{k}$$

$$\because (\vec{b_1} \times \vec{b_2}) \cdot (\vec{a_2} - \vec{a_1}) = 16 - 16 = 0$$

∴ The lines are intersecting and the shortest distance between the lines is 0.

Now for point of intersection

$$3\hat{\imath} + 2\hat{\jmath} - 4\hat{k} + \lambda(\hat{\imath} + 2\hat{\jmath} + 2\hat{k}) = 5\hat{\imath} - 2\hat{\jmath} + \mu(3\hat{\imath} + 2\hat{\jmath} + 6\hat{k})$$

$$\Rightarrow 3 + \lambda = 5 + 3\mu \qquad ----\qquad (1)$$
$$2 + 2\lambda = -2 + 2\mu \qquad ----\qquad (2)$$
$$-4 + 2\lambda = 6\mu \qquad ----\qquad (3)$$

Solving (1) ad (2) we get, $\mu = -2 \; and \; \lambda = -4$

Substituting in equation of line we get
$$\vec{r} = 5i - 2j + (-2)(3\hat{\imath} + 2\hat{\jmath} - 6\hat{k}) = -\hat{\imath} - 6\hat{\jmath} - 12\hat{k}$$

Point of intersection is $(-1, -6, -12)$

OR

Let P be the given point and Q be the foot of the perpendicular.

Equation of PQ $\frac{x+1}{2} = \frac{y-3}{1} = \frac{z+6}{-2} = \lambda$

P $(-1, 3, -6)$

Q
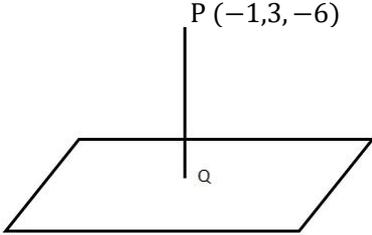
Let coordinates of Q be $(2\lambda - 1, \lambda + 3, -2\lambda - 6)$

Since Q lies in the plane $2x + y - 2z + 5 = 0$

$$\therefore 2(2\lambda - 1) + (\lambda + 3) - 2(-2\lambda - 6) + 5 = 0$$

$$\Rightarrow 4\lambda - 2 + \lambda + 3 + 4\lambda + 12 + 5 = 0$$

	1
	1
	1
	1
	1
	$1\frac{1}{2}$
	$\frac{1}{2}$

	$\Rightarrow 9\lambda + 18 = 0 \qquad \Rightarrow \lambda = -2$ $\therefore$ coordinates of Q are $(-5, \quad 1, -2)$ Length of the perpendicular $= \sqrt{(-5+1)^2 + (1-3)^2 + (-2+6)^2}$ $\qquad\qquad\qquad\qquad\qquad = 6\ units$		1 1 1
38	Max $Z = 3x + y$ Subject to $\quad\begin{aligned} x + 2y &\geq 100 \qquad\qquad ----- \qquad (1) \\ 2x - y &\leq 0 \qquad\qquad\quad ----- \qquad (2) \\ 2x + y &\leq 200 \qquad\qquad ----- \qquad (3) \\ x &\geq 0, \qquad y \geq 0 \end{aligned}$ 	Corner Points	$Z = 3x + y$	 |---|---| | A (0, 50) | 50 | | B (0, 200) | 200 | | C (50, 100) | 250 | | D (20, 40) | 100 | $Max\ z = 250 \quad at \quad x = 50, \qquad y = 100$		3 1 1

	OR	
(i)		

Corner points	$Z = 3x - 4y$
O(0,0)	0
A(0,8)	-32
B(4,10)	-28
C(6,8)	-14
D(6,5)	-2
E(4,0)	12

$$Max \quad Z = 12 \ at \ E(4,0)$$
$$Min \quad Z = -32 \ at \ A(0,8)$$

$1\frac{1}{2}$

1

(ii) Since maximum value of Z occurs at B(4,10) and C(6, 8)
∴ $4p + 10q = 6p + 8q$
$\Rightarrow 2q = 2p$
$\Rightarrow p = q$
Number of optimal solution are infinite

2
$\frac{1}{2}$