

## Marking Scheme

### Section A

1.  $\frac{1}{|AB|} = -\frac{1}{6} \Rightarrow \frac{1}{|A||B|} = -\frac{1}{6} \Rightarrow |B| = -3.$  [1]

2.  $2\sin(x^2)\cos(x^2)$  or  $\sin(2x^2)$  [1]

3. 2 [1]

4.  $l^2 + m^2 + n^2 = 1 \Rightarrow \left(\frac{1}{\sqrt{3}}\right)^2 + \left(\frac{1}{\sqrt{6}}\right)^2 + n^2 = 1 \Rightarrow \cos\gamma = \frac{1}{\sqrt{2}} \Rightarrow \gamma = 45^\circ$  or  $\frac{\pi}{4}$  [1]

OR

Direction ratios of the given line are 2, -1, 2. [1/2]

Hence, direction cosines of the line are:

$\frac{2}{3}, \frac{-1}{3}, \frac{2}{3}$  or  $\frac{-2}{3}, \frac{1}{3}, \frac{-2}{3}$  [1/2]

### Section B

5. An element  $(e, f) \in Z \times Z$  be the identity element, if  
 $(a, b) * (e, f) = (a, b) = (e, f) * (a, b) \quad \forall (a, b) \in Z \times Z$  [1/2]

i.e., if,  $(af + be, bf) = (a, b) = (eb + fa, fb)$

i.e., if,  $af + be = a = eb + fa$  and  $bf = b = fb$  ... (1)

i.e., if,  $f = 1, e = 0$  ... (2) [1]

Hence,  $(0, 1)$  is the identity element. [1/2]

6.  $A^2 = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$   
 $A^2 = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$  [1/2]

$5A = \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix}, \quad kI = \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$  [1/2]

$A^2 - 5A = kI$

$$\begin{bmatrix} -7 & 0 \\ 0 & -7 \end{bmatrix} = \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix} \Rightarrow k = -7 \quad [1]$$

$$7. \quad I = \int \left\{ \frac{(1+x^2) + (\sin^2 x - 1)}{1+x^2} \right\} \sec^2 x \, dx \quad [1/2]$$

$$\int \left[ 1 - \frac{\cos^2 x}{1+x^2} \right] \sec^2 x \, dx$$

$$\int \left[ \sec^2 x - \frac{1}{1+x^2} \right] dx \quad [1/2]$$

$$= \tan x - \tan^{-1} x + c \quad [1]$$

$$8. \quad \int \frac{e^x(x-3)}{(x-1)^3} dx$$

$$= \int e^x \left[ \frac{(x-1)-2}{(x-1)^3} \right] dx \quad [1/2]$$

$$= \int e^x \left[ \frac{1}{(x-1)^2} + \left( \frac{-2}{(x-1)^3} \right) \right] dx \quad (\because \int e^x [f(x) + f'(x)] dx = e^x f(x) + c) \quad [1/2]$$

$$= \frac{e^x}{(x-1)^2} + C \quad [1]$$

OR

$$I = \int \frac{(x^4 - x)^{\frac{1}{4}}}{x^5} dx$$

$$= \int \left[ x^4 \left( 1 - \frac{x}{x^4} \right) \right]^{\frac{1}{4}} \frac{1}{x^5} dx \quad \left[ \frac{1}{2} \right]$$

$$= \int x \left( 1 - \frac{1}{x^3} \right)^{\frac{1}{4}} \frac{1}{x^5} dx$$

$$= \int \left( 1 - \frac{1}{x^3} \right)^{\frac{1}{4}} \frac{1}{x^4} dx$$

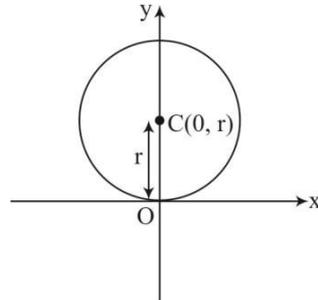
$$\text{Put } 1 - \frac{1}{x^3} = t \quad \left[ \frac{1}{2} \right]$$

So that,  $\frac{3}{x^4} dx = dt$

$$= \frac{1}{3} \int t^4 dt = \frac{1}{3} \cdot \frac{4}{5} t^{\frac{5}{4}} = \frac{4}{15} \left(1 - \frac{1}{x^3}\right)^{\frac{5}{4}} + C \quad [1]$$

9.  $(x-0)^2 + (y-r)^2 = r^2$  [1/2]

$$\Rightarrow x^2 + y^2 = 2ry \quad \dots(1)$$



Differentiating both sides w.r.t. x, we get

$$2x + 2yy' = 2ry'$$

$$\Rightarrow r = \frac{x + yy'}{y'} \quad \dots(2) \quad [1]$$

Substituting r from (2) in (1), we get

$$(x^2 + y^2)y' = 2y(x + yy') \quad [1/2]$$

10.  $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -3 & 4 \\ 2 & -1 & 2 \end{vmatrix} = -2\hat{i} + 4\hat{j} + 4\hat{k}$  [1]

$$|\vec{a} \times \vec{b}| = \sqrt{4+16+16} = 6 \quad [1/2]$$

$$\text{Area of the parallelogram} = \frac{|\vec{a} \times \vec{b}|}{2} = 3 \text{sq units.} \quad [1/2]$$

OR

The angle  $\theta$  between the vectors  $\vec{a}$  and  $\vec{b}$  is given by

$$\cos \theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|} \quad [1/2]$$

$$\text{i.e., } \cos \theta = \frac{(\hat{i} + \hat{j} - \hat{k}) \cdot (\hat{i} - \hat{j} + \hat{k})}{\sqrt{(1)^2 + (1)^2 + (-1)^2} \cdot \sqrt{(1)^2 + (-1)^2 + (1)^2}}$$

$$\text{i.e., } \cos \theta = \frac{1-1-1}{\sqrt{3} \cdot \sqrt{3}} \quad [1]$$

$$\text{i.e., } \cos \theta = -\frac{1}{3}$$

$$\Rightarrow \theta = \cos^{-1}\left(-\frac{1}{3}\right) \quad [1/2]$$

$$11. P(A' \cap B) = P(B) - P(A \cap B) \quad [1/2]$$

$$= P(B) - P(A) \cdot P(B) [\because A \text{ and } B \text{ are independent events}]$$

$$= (1 - P(A)) P(B)$$

$$= P(A') P(B) \quad [1]$$

$$\text{Since, } P(A' \cap B) = P(A') P(B)$$

Therefore  $A'$  and  $B$  are independent events. [1/2]

$$12. P(\text{Red transferred and red drawn or black transferred and red drawn})$$

$$= \frac{3}{8} \times \frac{7}{11} + \frac{5}{8} \times \frac{6}{11} \quad [1]$$

$$= \frac{51}{88} \quad [1]$$

OR

$$P(A|B) = 0.3$$

$$\Rightarrow \frac{P(A \cap B)}{P(B)} = 0.3$$

$$\Rightarrow P(A \cap B) = 0.5 \times 0.3 = 0.15 \quad [1]$$

$$\Rightarrow P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\Rightarrow P(A \cup B) = 0.6 + 0.5 - 0.15$$

$$\Rightarrow P(A \cup B) = 0.95 \quad [1]$$

### Section C

13. Let  $y \in \mathbb{R}$ , then for any  $x$ ,  $f(x) = y$  if  $y = 9x^2 + 6x - 5$

$$\Rightarrow y = (3x)^2 + 2(3x)(1) + (1)^2 - 5 - (1)^2$$

$$\Rightarrow y = (3x+1)^2 - 6$$

$$\Rightarrow 3x + 1 = \pm\sqrt{y+6}$$

$$\Rightarrow x = \frac{\pm\sqrt{y+6}-1}{3}$$

$$x = \frac{\sqrt{y+6}-1}{3} \left[ \because \frac{-\sqrt{y+6}-1}{3} \notin [0, \infty) \text{ for any value of } y \right]$$

Now, for  $y = -6 \in \mathbb{R}$ ,  $x = \frac{-1}{3} \notin [0, \infty)$

Hence,  $f(x)$  is not onto

$\Rightarrow f(x)$  is not invertible.

[1]

Since,  $x \geq 0$ , therefore  $\frac{\sqrt{y+6}-1}{3} \geq 0$

i.e.,  $\sqrt{y+6} \geq 1$

i.e.,  $y + 6 \geq 1$

$$y \geq -5$$

Redefining,  $f : [0, \infty) \rightarrow [-5, \infty)$

[1]

makes  $f(x) = 9x^2 + 6x - 5$  an onto function

Let  $x_1, x_2 \in [0, \infty)$  such that  $f(x_1) = f(x_2)$

$$\Rightarrow (3x_1 + 1)^2 = (3x_2 + 1)^2$$

$$\Rightarrow [3(x_1 + x_2) + 2] [3(x_1 - x_2)] = 0$$

$$\Rightarrow x_1 = x_2 \text{ (as } 3(x_1 + x_2) + 2 > 0)$$

$\therefore f(x)$  is one-one

[1]

Thus,  $f(x)$  is bijective, hence  $f$  is invertible

and  $f^{-1} : [-5, \infty) \rightarrow [0, \infty)$

$$f^{-1}(y) = \frac{\sqrt{y+6}-1}{3}$$

[1]

OR

**Reflexive:**

R is reflexive, as  $1 + a.a = 1 + a^2 > 0 \Rightarrow (a, a) \in R \quad \forall a \in \mathbf{R}$  [1]

**Symmetric:**

If  $(a, b) \in R$

then,  $1 + ab > 0$

$$\Rightarrow 1 + ba > 0$$

$$\Rightarrow (b, a) \in R$$

Hence, R is symmetric. [1]

**Transitive:**

$$\text{Let } a = -8, b = -1, c = \frac{1}{2}$$

$$\text{Since, } 1 + ab = 1 + (-8)(-1) = 9 > 0 \quad \therefore (a, b) \in R$$

$$\text{also, } 1 + bc = 1 + (-1)\left(\frac{1}{2}\right) = \frac{1}{2} > 0 \quad \therefore (b, c) \in R$$

$$\text{But, } 1 + ac = 1 + (-8)\left(\frac{1}{2}\right) = -3 < 0$$

Hence, R is not transitive. [2]

$$14. \sin\left(2 \tan^{-1} \frac{1}{4}\right) + \cos\left(\tan^{-1} 2\sqrt{2}\right)$$

$$\text{Let's evaluate, } \sin\left(2 \tan^{-1} \frac{1}{4}\right)$$

$$\text{Put } \tan^{-1} \frac{1}{4} = \theta$$

$$\Rightarrow \tan \theta = \frac{1}{4}$$

$$\text{Now, } \sin 2\theta = \frac{2 \tan \theta}{1 + \tan^2 \theta} = \frac{2 \times \frac{1}{4}}{1 + \left(\frac{1}{4}\right)^2} = \frac{8}{17} \quad \left[1\frac{1}{2}\right]$$

To evaluate  $\cos(\tan^{-1} 2\sqrt{2})$ , put  $\tan^{-1} 2\sqrt{2} = \phi$

$$\Rightarrow \tan \phi = 2\sqrt{2}$$

$$\Rightarrow \cos \phi = \frac{1}{3} \quad \left[ \frac{1}{2} \right]$$

$$\sin \left( 2 \tan^{-1} \frac{1}{4} \right) + \cos \left( \tan^{-1} 2\sqrt{2} \right) = \frac{8}{17} + \frac{1}{3} = \frac{41}{51} \quad [1]$$

15.

$$\begin{aligned} \text{LHS} &= \begin{vmatrix} a & b-c & c+b \\ a+c & b & c-a \\ a-b & b+a & c \end{vmatrix} C_1 \rightarrow aC_1 \\ &= \frac{1}{a} \begin{vmatrix} a^2 & b-c & c+b \\ a^2+ca & b & c-a \\ a^2-ab & b+a & c \end{vmatrix} C_1 \rightarrow C_1 + bC_2 + cC_3 \end{aligned} \quad [1]$$

$$\begin{aligned} &= \frac{1}{a} \begin{vmatrix} a^2+b^2+c^2 & b-c & c+b \\ a^2+b^2+c^2 & b & c-a \\ a^2+b^2+c^2 & b+a & c \end{vmatrix} \text{Taking } (a^2+b^2+c^2) \text{ common from } C_1 \\ & \quad [1] \end{aligned}$$

$$= \frac{(a^2+b^2+c^2)}{a} \begin{vmatrix} 1 & b-c & c+b \\ 1 & b & c-a \\ 1 & b+a & c \end{vmatrix} \begin{array}{l} R_2 \rightarrow R_2 - R_1 \\ R_3 \rightarrow R_3 - R_1 \end{array}$$

$$= \frac{(a^2+b^2+c^2)}{a} \begin{vmatrix} 1 & b-c & c+b \\ 0 & c & -a-b \\ 0 & a+c & -b \end{vmatrix} \quad \left[ \frac{1}{2} + \frac{1}{2} \right]$$

$$= \frac{(a^2+b^2+c^2)}{a} (-bc + a^2 + ac + ba + bc)$$

$$= (a^2+b^2+c^2)(a+b+c) = \text{R.H.S} \quad [1]$$

16. Let  $u = x^{\sin x}$

$$\Rightarrow \ln u = \sin x \ln x$$

$$\Rightarrow \frac{1}{u} \frac{du}{dx} = \frac{\sin x}{x} + \ln x \cos x$$

$$\Rightarrow \frac{du}{dx} = x^{\sin x} \left[ \frac{\sin x}{x} + \cos x \ln x \right] \quad \dots(1) \quad \left[ 1\frac{1}{2} \right]$$

Let  $v = \sin(x^x)$

Put  $t = x^x$

$$\log t = x \ln x$$

$$\frac{1}{t} \frac{dt}{dx} = x \frac{1}{x} + \ln x \cdot 1$$

$$\frac{dt}{dx} = x^x (1 + \ln x) \quad \dots(2)$$

$$\frac{dv}{dx} = \cos(x^x) \times \frac{dt}{dx} \text{ (where } t = x^x \text{)}$$

$$\frac{dv}{dx} = \cos(x^x) x^x (1 + \ln x) \quad \dots(3) \quad \text{(using (2))} \quad \left[ 1\frac{1}{2} \right]$$

Since,  $y = u + v$

Therefore,  $\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$

$$\frac{dy}{dx} = x^{\sin x} \left[ \frac{\sin x}{x} + \cos x \ln x \right] + x^x (1 + \ln x) \cos(x^x) \quad [1]$$

OR

$$y = \log(1 + 2t^2 + t^4)$$

$$y = \log(1 + t^2)^2$$

$$y = 2 \log(1 + t^2)$$

$$\frac{dy}{dt} = \frac{2(2t)}{1+t^2} = \frac{4t}{1+t^2} \quad \left[ 1\frac{1}{2} \right]$$

$$x = \tan^{-1} t$$

$$\frac{dx}{dt} = \frac{1}{1+t^2} \quad \left[ \frac{1}{2} \right]$$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{4t}{\frac{1}{1+t^2}} = 4t \quad [1]$$

$$\begin{aligned} \frac{d^2y}{dx^2} &= \frac{d}{dt}(4t) \times \frac{dt}{dx} \\ &= 4 \times (1+t^2) = 4(1+t^2) \quad [1] \end{aligned}$$

17.  $y = \cos(m \cos^{-1} x)$

$$\Rightarrow \frac{dy}{dx} = -\sin(m \cos^{-1} x) \times \frac{-m}{\sqrt{1-x^2}} \quad [1]$$

$$\Rightarrow \sqrt{1-x^2} \frac{dy}{dx} = m \sin(m \cos^{-1} x)$$

Squaring both sides, we get

$$\Rightarrow (1-x^2) \left( \frac{dy}{dx} \right)^2 = m^2 \sin^2(m \cos^{-1} x) \quad [1]$$

$$\Rightarrow (1-x^2) \left( \frac{dy}{dx} \right)^2 = m^2 (1-y^2) \quad \left[ \frac{1}{2} \right]$$

Differentiating again,

$$(1-x^2) 2 \left( \frac{dy}{dx} \right) \left( \frac{d^2y}{dx^2} \right) + \left( \frac{dy}{dx} \right)^2 (-2x) = m^2 \left( -2y \frac{dy}{dx} \right) \quad [1]$$

$$\Rightarrow (1-x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} + m^2 y = 0 \quad \left[ \frac{1}{2} \right]$$

18. The given curve is

$$y = 4x^3 - 3x + 5$$

Let the required normal be at  $(x_1, y_1)$

$$\text{Slope of the tangent} = \frac{dy}{dx} = 12x^2 - 3$$

$$m_1 = \text{slope of the normal} = \frac{-1}{\left(\frac{dy}{dx}\right)_{(x_1, x_2)}} = \frac{-1}{12x_1^2 - 3} \quad [1]$$

$$m_2 = \text{Slope of the line} = 9$$

Since normal is perpendicular to the line.

$$\text{Therefore, } m_1 \cdot m_2 = -1$$

$$\frac{-1}{12x_1^2 - 3} \times 9 = -1$$

$$\Rightarrow 12x_1^2 - 3 = 9$$

$$\Rightarrow x_1 = \pm 1$$

Hence, the points are (1, 6) and (-1, 4) [1]

Equations of normals are:

$$y - 6 = -\frac{1}{9}(x - 1) \text{ i.e., } x + 9y = 55 \quad [1]$$

$$\text{and } y - 4 = -\frac{1}{9}(x + 1) \text{ i.e., } x + 9y = 35 \quad [1]$$

$$19. I = \int \frac{x^4 + 1}{x(x^2 + 1)^2} dx$$

$$= \int \frac{(x^4 + 1)x}{x^2(x^2 + 1)^2} dx \quad \left[ \frac{1}{2} \right]$$

$$\text{Put } x^2 = t$$

$$\text{So that } 2x dx = dt$$

$$I = \frac{1}{2} \int \frac{(t^2 + 1) dt}{t(t + 1)^2} \quad \left[ \frac{1}{2} \right]$$

$$\text{Now, } \frac{t^2 + 1}{t(t + 1)^2} = \frac{A}{t} + \frac{B}{t + 1} + \frac{C}{(t + 1)^2}$$

$$t^2 + 1 = A(t + 1)^2 + Bt(t + 1) + Ct \quad \dots(1)$$

On comparing the coefficients of like terms in (1), we get

$$A = 1, B = 0, C = -2 \quad \left[ 1 \frac{1}{2} \right]$$

$$\begin{aligned}
 I &= \int \left[ \frac{1}{t} - \frac{2}{(t+1)^2} \right] dt \\
 &= \log |t| - \frac{2(t+1)^{-1}}{-1} + C \quad [1] \\
 &= 2 \log |x| + \frac{2}{x^2+1} + C \quad \left[ \frac{1}{2} \right]
 \end{aligned}$$

20.  $I = \int_{-1}^1 \frac{x+|x|+1}{x^2+2|x|+1} dx$

$$\begin{aligned}
 &= \int_{-1}^1 \frac{x}{x^2+2|x|+1} dx + \int_{-1}^1 \frac{|x|+1}{x^2+2|x|+1} dx \quad [1] \\
 &= I_1 + I_2 \text{ (say)} \quad \dots (1)
 \end{aligned}$$

Now,  $I_1 = \int_{-1}^1 \frac{x}{x^2+2|x|+1} dx$

Let  $f(x) = \frac{x}{x^2+2|x|+1}$

$$f(-x) = \frac{-x}{(-x)^2+2|-x|+1} = \frac{-x}{x^2+2|x|+1} = -f(x)$$

$\therefore f(x)$  is odd function.

Hence,  $I_1 = 0 \dots (2)$  [1]

Also,  $I_2 = \int_{-1}^1 \frac{|x|+1}{x^2+2|x|+1} dx$

Let  $g(x) = \frac{|x|+1}{x^2+2|x|+1} \Rightarrow g(-x) = \frac{|-x|+1}{(-x)^2+2|-x|+1}$

$$g(-x) = \frac{|x|+1}{x^2+2|x|+1} = g(x)$$

$\therefore g(x)$  is even function

$$\left[ \frac{1}{2} \right]$$

$\therefore I_2 = 2 \int_0^1 \frac{x+1}{x^2+2x+1} dx = 2 \int_0^1 \frac{1}{x+1} dx$

$$= 2[\ln|x+1|]_0^1 = 2[\ln 2 - \ln 1]$$

$$I_2 = 2\ln 2 \quad \dots(3) \quad [1]$$

From (1), (2) and (3), we get

$$I = 2\ln 2 \quad \left[ \frac{1}{2} \right]$$

21.  $\cos y dx + (1 + 2e^{-x}) \sin y dy = 0$

$$\Rightarrow \int \frac{dx}{1 + 2e^{-x}} = \int \frac{-\sin y}{\cos y} dy \quad [1]$$

$$\Rightarrow \int \frac{e^x}{2 + e^x} dx = \int \frac{-\sin y}{\cos y} dy$$

$$\Rightarrow \ln(e^x + 2) = \ln |\cos y| + \ln C$$

$$\Rightarrow \ln(e^x + 2) = \ln |\cos y| + C$$

$$\Rightarrow e^x + 2 = C \cos y \quad \dots(1) \quad \left[ \frac{1}{2} \right]$$

$$\Rightarrow e^x + 2 = \pm C \cos y \Rightarrow e^x + 2 = k \cos y$$

Substituting  $x = 0, y = \frac{\pi}{4}$  in (1), we get

$$1 + 2 = k \cos \frac{\pi}{4}$$

$$\Rightarrow k = 3\sqrt{2} \quad [1]$$

$$\therefore e^x + 2 = 3\sqrt{2} \cos y \text{ is the particular solution.} \quad \left[ \frac{1}{2} \right]$$

OR

$$\Rightarrow \frac{dx}{dy} = \frac{y \tan y - x \tan y - xy}{y \tan y}$$

$$\frac{dx}{dy} + \left( \frac{1}{y} + \frac{1}{\tan y} \right) x = 1 \quad [1]$$

$$\text{I.F} = e^{\int \left( \frac{1}{y} + \cot y \right) dy} = e^{\ln y + \ln \sin y}$$

$$I.F = e^{\ln(y \sin y)} = y \sin y \quad [1]$$

Solution of the D.E. is:

$$x \times I.F = \int (Q \times I.F) dy$$

$$\Rightarrow xy \sin y = \int y \sin y dy \quad [1]$$

$$\Rightarrow xy \sin y = y(-\cos y) - \int (-\cos y) dy$$

$$\Rightarrow xy \sin y = -y \cos y + \sin y + C$$

$$\Rightarrow x = \frac{\sin y - y \cos y + C}{y \sin y} \quad [1]$$

22. Let  $\vec{r} = a\hat{i} + b\hat{j} + c\hat{k}$  be the required vector.

Since,  $\vec{r} \perp \vec{q}$

$$\text{therefore, } 1a - 2b + 1c = 0 \quad \dots(1) \quad [1]$$

Also,  $\vec{p}, \vec{q}$  and  $\vec{r}$  are coplanar.

$$\therefore [\vec{p}, \vec{q}, \vec{r}] = 0$$

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & -2 & 1 \\ a & b & c \end{vmatrix} = 0 \Rightarrow a - c = 0 \quad \dots(2) \quad [1]$$

Solving equation (1) and (2)

$$\frac{a}{2-0} = \frac{b}{1+1} = \frac{c}{0+2}$$

$$\Rightarrow \frac{a}{2} = \frac{b}{2} = \frac{c}{2}$$

$$\text{i.e., } \frac{a}{1} = \frac{b}{1} = \frac{c}{1}$$

$$\therefore \vec{r} = 1\hat{i} + 1\hat{j} + 1\hat{k} \quad [1]$$

$$|\vec{r}| = \sqrt{3}$$

$$\therefore \text{Unit vector } \hat{r} = \frac{\vec{r}}{|\vec{r}|} = \frac{\hat{i} + \hat{j} + \hat{k}}{\sqrt{3}}$$

$$\therefore \text{Required vector} = 5\sqrt{3} \hat{r} = 5(\hat{i} + \hat{j} + \hat{k}) \quad [1]$$

23. Vector equation of the line passing through

(1, 2, 3) and (-3, 4, 3) is

$\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a})$  where

$$\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k} \text{ and } \vec{b} = -3\hat{i} + 4\hat{j} + 3\hat{k}$$

$$\Rightarrow \vec{r} = \hat{i} + 2\hat{j} + 3\hat{k} + \lambda(-4\hat{i} + 2\hat{j}) \quad \dots(1) \quad [2]$$

Equation of z-axis is

$$\vec{r} = \mu\hat{k} \quad \dots(2) \quad [1]$$

Since  $(-4\hat{i} + 2\hat{j}) \cdot \hat{k} = 0$

$\therefore$  line (1) is  $\perp$  to z-axis. [1]

### Section D

$$24. A = \begin{bmatrix} 3 & 1 & 2 \\ 3 & 2 & -3 \\ 2 & 0 & -1 \end{bmatrix}$$

$$|A| = 3(-2) - 1(3) + 2(-4) = -6 - 3 - 8 = -17 \neq 0 \quad [1]$$

$\therefore A^{-1}$  exists. [1/2]

$$\text{Cofactor matrix of } A = \begin{bmatrix} -2 & -3 & -4 \\ 1 & -7 & 2 \\ -7 & 15 & 3 \end{bmatrix} \quad [2]$$

$$A^{-1} = \frac{1}{|A|} \text{Adj } A = \frac{1}{-17} \begin{bmatrix} -2 & 1 & -7 \\ -3 & -7 & 15 \\ -4 & 2 & 3 \end{bmatrix} \quad \left[ \frac{1}{2} \right]$$

Now for given system of equations.

$$\begin{bmatrix} 3 & 3 & 2 \\ 1 & 2 & 0 \\ 2 & -3 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \\ 5 \end{bmatrix}$$

$$(A^t)X = B$$

$$\Rightarrow X = (A^t)^{-1}B$$

$$\Rightarrow X = (A^{-1})^t B \quad [ \because (A^t)^{-1} = (A^{-1})^t ] \quad \left[ \begin{array}{c} 1 \\ 2 \end{array} \right]$$

$$X = \frac{1}{-17} \begin{bmatrix} -2 & -3 & -4 \\ 1 & -7 & 2 \\ -7 & 15 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 4 \\ 5 \end{bmatrix}$$

$$X = \frac{1}{-17} \begin{bmatrix} -34 \\ -17 \\ 68 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ -4 \end{bmatrix}$$

$$x = 2, y = 1, z = -4 \quad \left[ \begin{array}{c} 1 \\ 2 \end{array} \right]$$

OR

$$A = IA \quad \left[ \begin{array}{c} 1 \\ 2 \end{array} \right]$$

$$\begin{bmatrix} 2 & -1 & 3 \\ -5 & 3 & 1 \\ -3 & 2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \quad R_1 \rightarrow R_1 + R_3 \quad \left[ \begin{array}{c} 1 \\ 2 \end{array} \right]$$

$$\begin{bmatrix} -1 & 1 & 6 \\ -5 & 3 & 1 \\ -3 & 2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \quad R_1 \rightarrow (-1)R_1 \quad \left[ \begin{array}{c} 1 \\ 2 \end{array} \right]$$

$$\begin{bmatrix} 1 & -1 & -6 \\ -5 & 3 & 1 \\ -3 & 2 & 3 \end{bmatrix} = \begin{bmatrix} -1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \quad \begin{array}{l} R_2 \rightarrow R_2 + 5R_1 \\ R_3 \rightarrow R_3 + 3R_1 \end{array} \quad [1]$$

$$\begin{bmatrix} 1 & -1 & -6 \\ 0 & -2 & -29 \\ 0 & -1 & -15 \end{bmatrix} = \begin{bmatrix} -1 & 0 & -1 \\ -5 & 1 & -5 \\ -3 & 0 & -2 \end{bmatrix} A \quad R_2 \leftrightarrow R_3 \quad \left[ \begin{array}{c} 1 \\ 2 \end{array} \right]$$

$$\begin{bmatrix} 1 & -1 & -6 \\ 0 & -1 & -15 \\ 0 & -2 & -29 \end{bmatrix} = \begin{bmatrix} -1 & 0 & -1 \\ -3 & 0 & -2 \\ -5 & 1 & -5 \end{bmatrix} A \quad R_2 \rightarrow (-1)R_2 \quad \left[ \begin{array}{c} 1 \\ 2 \end{array} \right]$$

$$\begin{bmatrix} 1 & -1 & -6 \\ 0 & 1 & 15 \\ 0 & -2 & -29 \end{bmatrix} = \begin{bmatrix} -1 & 0 & -1 \\ 3 & 0 & 2 \\ -5 & 1 & -5 \end{bmatrix} A \quad \begin{array}{l} R_1 \rightarrow R_1 + R_2 \\ R_3 \rightarrow R_3 + 2R_2 \end{array} \quad [1]$$

$$\begin{bmatrix} 1 & 0 & 9 \\ 0 & 1 & 15 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 1 \\ 3 & 0 & 2 \\ 1 & 1 & -1 \end{bmatrix} A \quad \begin{array}{l} R_1 \rightarrow R_1 - 9R_3 \\ R_2 \rightarrow R_2 - 15R_3 \end{array} \quad [1]$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -7 & -9 & 10 \\ -12 & -15 & 17 \\ 1 & 1 & -1 \end{bmatrix} A$$

$$\text{Hence, } A^{-1} = \begin{bmatrix} -7 & -9 & 10 \\ -12 & -15 & 17 \\ 1 & 1 & -1 \end{bmatrix} \quad \left[ \frac{1}{2} \right]$$

25. Let the length and breadth of the base = x.

Also let the height of the godown = y.

Let C be the cost of constructing the godown and V be the given volume.  $\left[ \frac{1}{2} \right]$

Since cost is proportional to the area, therefore

$$C = k[3x^2 + 4xy], \text{ where } k > 0 \text{ is constant of proportionality} \quad \dots(1) \quad \left[ \frac{1}{2} \right]$$

$$x^2y = V(\text{constant}) \quad \dots(2) \quad \left[ \frac{1}{2} \right]$$

$$y = \frac{V}{x^2} \quad \dots(3)$$

Substituting value of y from equation (3), in equation (1), we get

$$C = k \left[ 3x^2 + 4x \left( \frac{V}{x^2} \right) \right] = k \left[ 3x^2 + \frac{4V}{x} \right]$$

$$\frac{dC}{dx} = k \left[ 6x - \frac{4V}{x^2} \right] \quad \dots(1) \quad [1]$$

For maximum or minimum value of S

$$\frac{dC}{dx} = 0$$

$$\Rightarrow 6x - \frac{4V}{x^2} = 0$$

$$\Rightarrow x = \left(\frac{2V}{3}\right)^{\frac{1}{3}} \quad [1]$$

$$\text{when, } x = \left(\frac{2V}{3}\right)^{\frac{1}{3}}, \frac{d^2C}{dx^2} = 6 + \frac{8V}{x^3} = 18 > 0 \quad [1]$$

$$\therefore \text{C is minimum when } x = \left(\frac{2V}{3}\right)^{\frac{1}{3}} \text{ and } y = \frac{(18V)^{\frac{1}{3}}}{2} \quad \left[\frac{1}{2}\right]$$

26. The given curves are

$$y = \sqrt{x} \quad \dots(1)$$

$$2y + 3 = x \quad \dots(2)$$

Solving equation (1) and (2), we get

$$2y + 3 = y^2$$

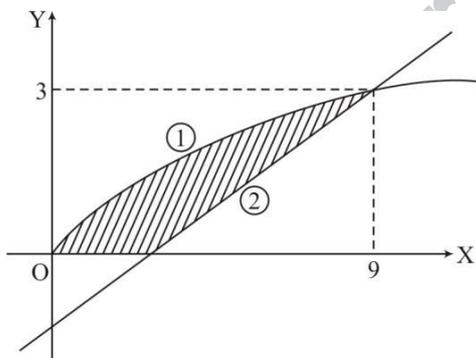
$$\Rightarrow y = -1, 3$$

$$\Rightarrow y = 3 \text{ (as } y > 0)$$

Substituting value of  $y = 3$  in (2) we get

$$x = 2(3) + 3 = 9$$

i.e., (1) and (2) intersects at (9, 3) [1]



$$\left[\frac{1}{2}\right]$$

$$\text{Required Area} = \int_0^3 (2y + 3) dy - \int_0^3 y^2 dy \quad \left[\frac{1}{2}\right]$$

$$= \left[ y^2 + 3y - \frac{y^3}{3} \right]_0^3 \quad [1]$$

$$= 9 + 9 - 9$$

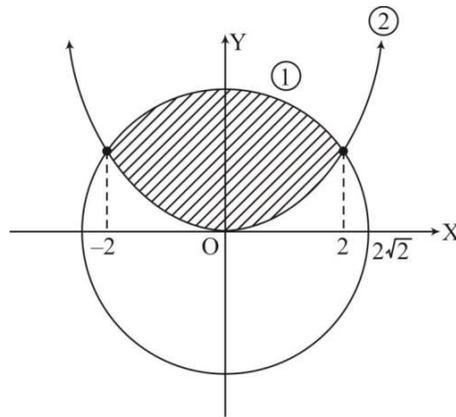
$$= 9 \text{ sq units.} \quad [1]$$

OR

The given curves are

$$x^2 + y^2 = 8 \quad \dots(1)$$

$$x^2 = 2y \quad \dots(2)$$



$$\left[ 1 \frac{1}{2} \right]$$

Solving (1) and (2)

$$8 - y^2 = 2y \Rightarrow y = 2, -4 \Rightarrow y = 2 \text{ (as } y > 0)$$

$$\text{Substituting } y = 2 \text{ in (2) we get } x^2 = 4 \Rightarrow x = -2 \text{ or } 2 \quad [1]$$

$$\text{Required Area} = \int_{-2}^2 \sqrt{8 - x^2} dx - \int_{-2}^2 \frac{x^2}{2} dx \quad [1]$$

$$= 2 \left[ \int_0^2 \sqrt{(2\sqrt{2})^2 - x^2} dx - \int_0^2 \frac{x^2}{2} dx \right]$$

$$= 2 \left[ \frac{x}{2} \sqrt{8 - x^2} + \frac{8}{2} \sin^{-1} \left( \frac{x}{2\sqrt{2}} \right) \right]_0^2 - \frac{1}{3} [x^3]_0^2 \quad \left[ 1 \frac{1}{2} \right]$$

$$\begin{aligned}
 &= 2 \left[ 2 + 4 \left( \frac{\pi}{4} \right) - 0 \right] - \frac{1}{3} [8 - 0] \\
 &= 4 + 2\pi - \frac{8}{3} \\
 &= \left( 2\pi + \frac{4}{3} \right) \text{sq. units} \quad [1]
 \end{aligned}$$

27. The two given lines are

$$\frac{x-1}{3} = \frac{y-4}{2} = \frac{z-4}{-2} \quad \dots(1)$$

and  $\frac{x+1}{2} = \frac{y-1}{-4} = \frac{z+2}{1} \quad \dots(2)$

Let a, b, c be the D.R's of the normal to the plane containing the line (1). Therefore, equation of plane is

$$a(x-1) + b(y-4) + c(z-4) = 0 \quad \dots(3) \quad [1]$$

$$3a + 2b - 2c = 0 \quad \dots(4) \quad (\because \text{Required plane contains line (1)})$$

$$2a - 4b + 1c = 0 \quad \dots(5) \quad (\because \text{line (2) is parallel to the required plane}) \quad \left[ \frac{1}{2} + \frac{1}{2} \right]$$

$$\Rightarrow \frac{a}{2-8} = \frac{b}{-4-3} = \frac{c}{-12-4}$$

$$\Rightarrow \frac{a}{6} = \frac{b}{7} = \frac{c}{16} = \lambda$$

Putting,  $a = 6\lambda$ ,  $b = 7\lambda$ ,  $c = 16\lambda$  in (3), we get [1]

$$\Rightarrow 6\lambda(x-1) + 7\lambda(y-4) + 16\lambda(z-4) = 0$$

$$\Rightarrow 6x + 7y + 16z - 98 = 0, \text{ which is the required equation of the plane} \quad [1]$$

Since line (2) is parallel to required plane

$\therefore$  SD between two lines = Perpendicular distance of the point  $(-1, 1, -2)$  from the plane.

$$\text{i.e., } SD = \left| \frac{6(-1) + 7(1) + 16(-2) - 98}{\sqrt{6^2 + 7^2 + 16^2}} \right|$$

$$SD = \frac{129}{\sqrt{341}} \text{ units} \quad [2]$$

OR

The given line  $\frac{x+1}{1} = \frac{y+1}{2} = \frac{z+1}{3}$  ... (1) is coplanar with the line determined by the planes  $x+2y+3z-8=0$  ... (2) and  $2x+3y+4z-11=0$  ... (3), if we are able to show there exists a plane passing through intersection of planes (2) and (3) containing the line (1).

Equation of the plane passing through the intersection of planes (2) and (3) is

$$(x+2y+3z-8) + k(2x+3y+4z-11) = 0 \quad \dots(4) \quad [2]$$

We find, value of k for which the plane given by (4) passes through the point  $(-1, -1, -1)$  lying on line (1).

Substituting the coordinates of the point  $(-1, -1, -1)$  in (4), we get

$$(-1-2-3-8) + k(-2-3-4-11) = 0$$

$$-14 - 20k = 0$$

$$k = -\frac{7}{10}$$

Putting,  $k = -\frac{7}{10}$  in (4) we get

$$(x+2y+3z-8) - \frac{7}{10}(2x+3y+4z-11) = 0$$

$$\Rightarrow 4x + y - 2z + 3 = 0 \quad \dots(5) \quad [2]$$

Now we find value of  $a_1a_2 + b_1b_2 + c_1c_2$ , where  $a_1, b_1, c_1$  and D. Ratios of the line (1) and  $a_2, b_2, c_2$  and D. Ratios of the normal to the plane (5)

$$a_1a_2 + b_1b_2 + c_1c_2 = 1(4) + 2(1) + 3(-2)$$

$$\text{i.e., } a_1a_2 + b_1b_2 + c_1c_2 = 0 \quad [1]$$

which implies line (1) lies in plane (5)

Hence the two lines are coplanar and the equation of the plane containing them is

$$4x + y - 2z + 3 = 0 \quad [1]$$

28. Let the manufacturer make x and y quantity of toy A and toy B respectively.

$$\text{Max } P = 50x + 60y \quad [1]$$

Subject to

$$20x + 10y \leq 180 \quad \dots(1)$$

$$10x + 20y \leq 120 \quad \dots(2)$$

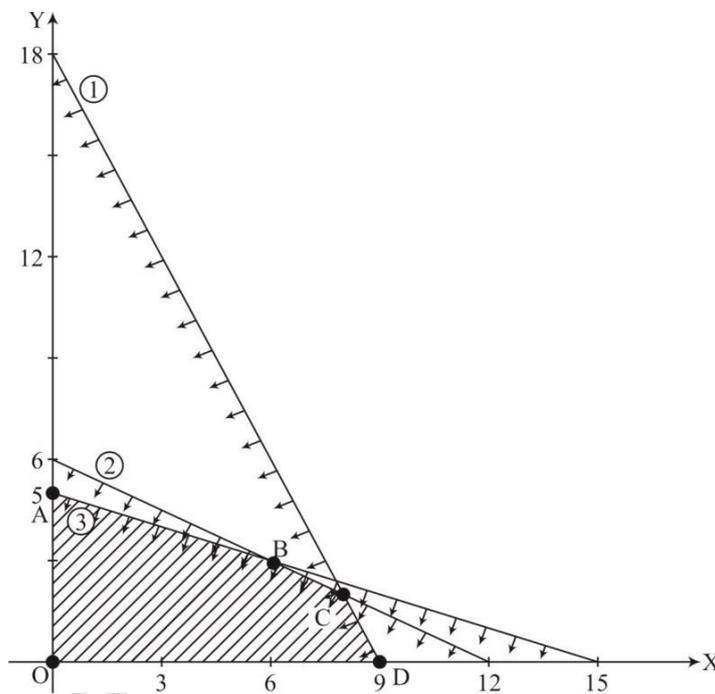
$$\left[ \frac{1}{2} \right]$$

$$10x + 30y \leq 150 \quad \dots(3)$$

$$x, y \geq 0$$

Corner Points	$P = 50x + 60y$
O (0, 0)	0
A (0, 5)	300
B (6, 3)	480
C (8, 2)	520 (Max)
D (9, 0)	450

[1]



[2]

Hence, Max Profit is ₹ 520, at  $x = 8$  and  $y = 2$

$$\left[ \frac{1}{2} \right]$$

29. Let A be the event that car delivered to firm needs service and tuning. Also let  $E_1, E_2$  and

$E_3$  be the events of car being rented from agencies X, Y and Z respectively.

$$\left[ \frac{1}{2} \right]$$

$$P(E_1) = \frac{50}{100} \quad P(E_2) = \frac{30}{100} \quad P(E_3) = \frac{20}{100} \quad [1]$$

$$P(A|E_1) = \frac{9}{100} \quad P(A|E_2) = \frac{12}{100} \quad P(A|E_3) = \frac{10}{100} \quad \left[1\frac{1}{2}\right]$$

$$P(E_3|A) = \frac{P(E_3)P(A|E_3)}{P(E_1)P(A|E_1) + P(E_2)P(A|E_2) + P(E_3)P(A|E_3)} \quad [1]$$

$$\Rightarrow P(E_3|A) = \frac{\frac{20}{100} \times \frac{10}{100}}{\frac{50}{100} \times \frac{9}{100} + \frac{30}{100} \times \frac{12}{100} + \frac{20}{100} \times \frac{10}{100}} = \frac{20}{101} \quad [1]$$

$$P(E'_3|A) = 1 - P(E_3|A) = 1 - \frac{20}{101} = \frac{81}{101} \quad [1]$$

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