SECTION: A (Solution of MCQs of 1 Mark each)

| Q no.	ANS	HINTS/SOLUTION
1.	(D)	For a square matrix A of order $n \times n$, we have $A.(adj\,A) = \|A\|I_n$, where I_n is the identity matrix of order $n \times n$. So, $A.(adj\,A) = \begin{bmatrix} 2025 & 0 & 0 \\ 0 & 2025 & 0 \\ 0 & 0 & 2025 \end{bmatrix} = 2025 I_3 \Rightarrow \|A\| = 2025$ & $\|adj\,A\| = \|A\|^{3-1} = (2025)^2$ $\therefore \|A\| + \|adj\,A\| = 2025 + (2025)^2$.
2.	(A)	$P \downarrow \text{Order}\ p \times k \qquad Y \downarrow \text{Order}\ 3 \times k \qquad W \downarrow \text{Order}\ n \times 3 \qquad Y \downarrow \text{Order}\ 3 \times k$ For PY to exist $k = 3$ Order of WY $= n \times k$ Order of $PY = p \times k$ For $PY + WY$ to exist order $(PY) = $ order (WY) $\therefore p = n$
3.	(C)	$y = e^x \Rightarrow \dfrac{dy}{dx} = e^x$ In the domain (R) of the function $\dfrac{dy}{dx} > 0$ hence the function is strictly increasing in $(-\infty, \infty)$
4.	(B)	$\|A\| = 5, \ \|B^{-1}AB\|^2 = (\|B^{-1}\|\|A\|\|B\|)^2 = \|A\|^2 = 5^2$.
5.	(B)	A differential equation of the form $\dfrac{dy}{dx} = f(x, y)$ is said to be homogeneous, if $f(x, y)$ is a homogeneous function of degree 0. Now, $x^n \dfrac{dy}{dx} = y\left(\log_e \dfrac{y}{x} + \log_e e\right) \Rightarrow \dfrac{dy}{dx} = \dfrac{y}{x^n}\left(\log_e e.\left(\dfrac{y}{x}\right)\right) = f(x, y); (Let).$ $f(x, y)$ will be a homogeneous function of degree **0,** if $n = 1$.
6.	(A)	**Method 1:** (*Short cut*) When the points $(x_1, y_1), (x_2, y_2)$ and $(x_1 + x_2, y_1 + y_2)$ are collinear in the Cartesian plane then $\begin{vmatrix} x_1 - x_2 & y_1 - y_2 \\ x_1 - (x_1 + x_2) & y_1 - (y_1 + y_2) \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} x_1 - x_2 & y_1 - y_2 \\ -x_2 & -y_2 \end{vmatrix} = (-x_1 y_2 + x_2 y_2 + x_2 y_1 - x_2 y_2) = 0$ $\Rightarrow x_2 y_1 = x_1 y_2$.

		Method 2: When the points $(x_1, y_1), (x_2, y_2)$ and $(x_1 + x_2, y_1 + y_2)$ are collinear in the Cartesian plane then $$\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_1 + x_2 & y_1 + y_2 & 1 \end{vmatrix} = 0$$ $\Rightarrow 1.\left(x_2 y_1 + x_2 y_2 - x_1 y_2 - x_2 y_2\right) - 1\left(x_1 y_1 + x_1 y_2 - x_1 y_1 - x_2 y_1\right) + \left(x_1 y_2 - x_2 y_1\right) = 0$ $\Rightarrow x_2 y_1 = x_1 y_2.$
7.	(A)	$A = \begin{bmatrix} 0 & 1 & c \\ -1 & a & -b \\ 2 & 3 & 0 \end{bmatrix}$ When the matrix A is skew symmetric then $A^T = -A \Rightarrow a_{ij} = -a_{ji}$; $\Rightarrow c = -2; a = 0$ and $b = 3$ So , $a + b + c = 0 + 3 - 2 = 1.$
8.	(C)	$P\left(\overline{A}\right) = \dfrac{1}{2}; P\left(\overline{B}\right) = \dfrac{2}{3}; P\left(A \cap B\right) = \dfrac{1}{4}$ $\Rightarrow P(A) = \dfrac{1}{2}; P(B) = \dfrac{1}{3}$ **We have,** $P\left(A \cup B\right) = P\left(A\right) + P\left(B\right) - P\left(A \cap B\right) = \dfrac{1}{2} + \dfrac{1}{3} - \dfrac{1}{4} = \dfrac{7}{12}$ $P\left(\dfrac{\overline{A}}{\overline{B}}\right) = \dfrac{P\left(\overline{A} \cap \overline{B}\right)}{P\left(\overline{B}\right)} = \dfrac{P\overline{\left(A \cup B\right)}}{P\left(\overline{B}\right)} = \dfrac{1 - P\left(A \cup B\right)}{P\left(\overline{B}\right)} = \dfrac{1 - \dfrac{7}{12}}{\dfrac{2}{3}} = \dfrac{5}{8}.$
9.	(B)	For obtuse angle, $\cos\theta < 0 \implies \vec{p}.\vec{q} < 0$ $2\alpha^2 - 3\alpha + \alpha < 0 \implies 2\alpha^2 - 2\alpha < 0 \implies \alpha \in (0, 1)$
10.	(C)	$\left|\vec{a}\right| = 3, \left|\vec{b}\right| = 4, \left|\vec{a} + \vec{b}\right| = 5$ We have , $\left|\vec{a} + \vec{b}\right|^2 + \left|\vec{a} - \vec{b}\right|^2 = 2\left(\left|\vec{a}\right|^2 + \left|\vec{b}\right|^2\right) = 2(9 + 16) = 50 \Rightarrow \left|\vec{a} - \vec{b}\right| = 5.$
11.	(C)	Since, the feasible region is bounded so the maximum and minimum values of the objective function will always exist.
12.	(A)	$\displaystyle\int \dfrac{dx}{x^3(1 + x^4)^{\frac{1}{2}}} = \int \dfrac{dx}{x^5\left(1 + \frac{1}{x^4}\right)^{\frac{1}{2}}}$ (Let $1 + x^{-4} = 1 + \dfrac{1}{x^4} = t, dt = -4x^{-5}dx = -\dfrac{4}{x^5}dx \Rightarrow \dfrac{dx}{x^5} = -\dfrac{1}{4}dt$) $= -\dfrac{1}{4}\int \dfrac{dt}{t^{\frac{1}{2}}} = -\dfrac{1}{4} \times 2 \times \sqrt{t} + c$, where $'c'$ denotes any arbitrary constant of integration. $= -\dfrac{1}{2}\sqrt{1 + \dfrac{1}{x^4}} + c = -\dfrac{1}{2x^2}\sqrt{1 + x^4} + c$

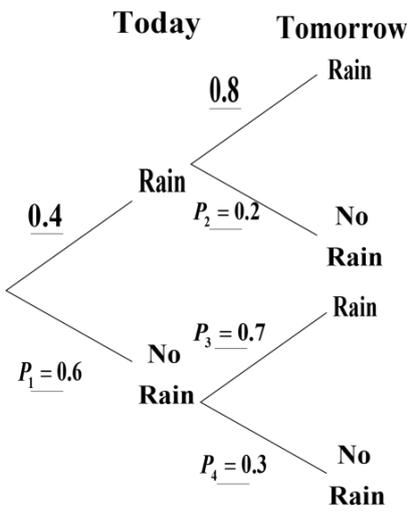| 13.	(A)	We know, $\int_0^{2a} f(x)dx = 0$, if $f(2a - x) = -f(x)$ Let $f(x) = cosec^7 x$. Now, $f(2\pi - x) = cosec^7(2\pi - x) = -cosec^7 x = -f(x)$ $\therefore \int_0^{2\pi} cosec^7 x \, dx = 0$; Using the property $\int_0^{2a} f(x)dx = 0$, if $f(2a - x) = -f(x)$.
14.	(B)	The given differential equation $e^{y'} = x \implies \frac{dy}{dx} = \log x$ $dy = \log x \, dx \implies \int dy = \int \log x \, dx$ $y = x \log x - x + c$ Hence, the correct option is **(B)**.
15.	(B)	Domain of $\cos^{-1} x$ is $[-1,1]$. and $\sqrt{x+1}$ is non-negative $\therefore 0 \le \sqrt{x+1} \le 1$ $\implies 0 \le x + 1 \le 1 \implies -1 \le x \le 0$ $\therefore$ domain of f(x) is $[-1, 0]$.
16.	(A)	**Since the given point $(-3, 2)$ satisfies the strict inequality therefore the region will contain this point and also it is an open half plane.**
17.	(D)	For the function $f: R \to R$ defined by $f(x) = [x]$; LHL = RHL at x = 2.5 Also LHD = RHD at x = 2.5. Hence, the function is continuous and differentiable at **$x = 2.5$**.
18.	(B)	**The required region is symmetric about the y – axis.** So, required area is (in sq. units) $= \left\| 2\int_0^4 2\sqrt{y} dy \right\| = 4\left[\frac{y^{\frac{3}{2}}}{\frac{3}{2}} \right]_0^4 = \frac{64}{3}$.
19.	(A)	**Both (A) and (R) are true and (R) is the correct explanation of (A).**
20.	(A)	**Both (A) and (R) are true and (R) is the correct explanation of (A).**

Section –B

[This section comprises of solution of very short answer type questions (VSA) of 2 marks each]

21	$\cot^{-1}(3x+5) > \dfrac{\pi}{4} = \cot^{-1} 1$	$\dfrac{1}{2}$
	$=> 3x + 5 < 1$ (as $\cot^{-1}x$ is strictly decreasing function in its domain)	$\dfrac{1}{2}$
	$=> 3x < -4$	
	$=> x < -\dfrac{4}{3}$	
	$\therefore x \in \left(-\infty, -\dfrac{4}{3}\right)$	1
22.	The marginal cost function is $C'\left(x\right) = 0.00039x^2 + 0.004x + 5$.	1
	$C'\left(150\right) = ₹ \ 14.375$.	1
23.(a)	$y = \tan^{-1} x$ and $z = \log_e x$.	
	Then $\dfrac{dy}{dx} = \dfrac{1}{1+x^2}$	$\dfrac{1}{2}$
	and $\dfrac{dz}{dx} = \dfrac{1}{x}$	$\dfrac{1}{2}$
	So, $\dfrac{dy}{dz} = \dfrac{\dfrac{dy}{dx}}{\dfrac{dz}{dx}}$	$\dfrac{1}{2}$
	$= \dfrac{\dfrac{1}{1+x^2}}{\dfrac{1}{x}} = \dfrac{x}{1+x^2}$.	$\dfrac{1}{2}$
OR 23.(b)	Let $y = (\cos x)^x$. Then, $y = e^{x\log_e\cos x}$	
	On differentiating both sides with respect to x, we get $\dfrac{dy}{dx} = e^{x\log_e\cos x}\dfrac{d}{dx}(x\log_e\cos x)$	$\dfrac{1}{2}$
	$\Rightarrow \dfrac{dy}{dx} = (\cos x)^x\left\{\log_e\cos x\dfrac{d}{dx}(x) + x\dfrac{d}{dx}(\log_e\cos x)\right\}$	$\dfrac{1}{2}$
	$\Rightarrow \dfrac{dy}{dx} = (\cos x)^x\left\{\log_e\cos x + x.\dfrac{1}{\cos x}(-\sin x)\right\} \Rightarrow \dfrac{dy}{dx} = (\cos x)^x(\log_e\cos x - x\tan x)$.	1
24.(a)	We have $\vec{b} + \lambda\vec{c} = (-1 + 3\lambda)\hat{\imath} + (2 + \lambda)\hat{\jmath} + \hat{k}$	$\dfrac{1}{2}$
	$(\vec{b} + \lambda\vec{c}).\vec{a} = 0 \ => \ 2(-1 + 3\lambda) + 2(2 + \lambda) + 3 = 0$	1
	$\lambda = -\dfrac{5}{8}$	$\dfrac{1}{2}$
OR 24.(b)	$\overrightarrow{BA} = \overrightarrow{OA} - \overrightarrow{OB} = \left(4\hat{\imath} + 3\hat{k}\right) - \hat{k} = 4\hat{\imath} + 2\hat{k}$	$\dfrac{1}{2}$

	$\widehat{BA} = \dfrac{4}{2\sqrt{5}}\hat{\imath} + \dfrac{2}{2\sqrt{5}}\hat{k} = \dfrac{2}{\sqrt{5}}\hat{\imath} + \dfrac{1}{\sqrt{5}}\hat{k}$	$\dfrac{1}{2}$
	So, the angles made by the vector $\overrightarrow{BA}$ with the x, y and the z axes are respectively $\cos^{-1}\left(\dfrac{2}{\sqrt{5}}\right), \dfrac{\pi}{2}, \cos^{-1}\left(\dfrac{1}{\sqrt{5}}\right).$	1
25.	$\overrightarrow{d_1} = \vec{a} + \vec{b} = 4\hat{\imath} - 2\hat{\jmath} - 2\hat{k}$, $\overrightarrow{d_2} = \vec{a} - \vec{b} = -6\hat{\jmath} - 8\hat{k}$	$\dfrac{1}{2}$
	Area of the parallelogram $= \dfrac{1}{2}\left|\overrightarrow{d_1} \times \overrightarrow{d_2}\right| = \dfrac{1}{2}\begin{Vmatrix}\hat{\imath} & \hat{\jmath} & \hat{k} \\ 4 & -2 & -2 \\ 0 & -6 & -8\end{Vmatrix} = 2\left|\hat{\imath} + 8\hat{\jmath} - 6\hat{k}\right|$	1
	Area of the parallelogram $= 2\sqrt{101}$ sq. units.	$\dfrac{1}{2}$

Section –C

[This section comprises of solution short answer type questions (SA) of 3 marks each]
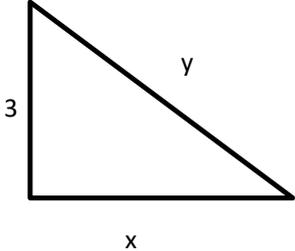
26.	$$x^2 + 3^2 = y^2$$ $$When\ y = 5\ then\ x = 4, now\ 2x\dfrac{dx}{dt} = 2y\dfrac{dy}{dt}$$ $$4(200) = 5\dfrac{dy}{dt} \Rightarrow \dfrac{dy}{dt} = 160 \text{ cm/s}$$	1 1 1
27.	$A = \dfrac{1}{3}\sqrt{t}$ $\therefore \dfrac{dA}{dt} = \dfrac{1}{6}t^{-\frac{1}{2}} = \dfrac{1}{6\sqrt{t}};\ \ \forall t \in (5,18)$	1
	$\dfrac{dA}{dt} = \dfrac{1}{6\sqrt{t}} \therefore \dfrac{d^2A}{dt^2} = -\dfrac{1}{12t\sqrt{t}}$	1
	So, $\dfrac{d^2A}{dt^2} < 0,\ \forall t \in (5,18)$	$\dfrac{1}{2}$
	This means that the rate of change of the ability to understand spatial concepts decreases (slows down) with age.	$\dfrac{1}{2}$
28(a) (i)	$\theta = \cos^{-1}\left(\dfrac{\overrightarrow{l_1}.\overrightarrow{l_2}}{|\overrightarrow{l_1}|.|\overrightarrow{l_2}|}\right) = \cos^{-1}\left(\dfrac{(\hat{\imath}-2\hat{\jmath}+3\hat{k}).(3\hat{\imath}-2\hat{\jmath}+\hat{k})}{|(\hat{\imath}-2\hat{\jmath}+3\hat{k})||(3\hat{\imath}-2\hat{\jmath}+\hat{k})|}\right)$	1
	$= \cos^{-1}\left(\dfrac{3+4+3}{\sqrt{1+4+9}\sqrt{9+4+1}}\right) = \cos^{-1}\left(\dfrac{10}{14}\right) = \cos^{-1}\left(\dfrac{5}{7}\right).$	$\dfrac{1}{2}$

	(ii) Scalar projection of $\vec{l_1}$ on $\vec{l_2}$ $= \dfrac{\vec{l_1}.\vec{l_2}}{|\vec{l_2}|} = \dfrac{(\hat{\imath}-2\hat{\jmath}+3\hat{k}).(3\hat{\imath}-2\hat{\jmath}+\hat{k})}{|(3\hat{\imath}-2\hat{\jmath}+\hat{k})|}$		1
	$= \dfrac{3+4+3}{\sqrt{9+4+1}} = \dfrac{10}{\sqrt{14}}.$		$\dfrac{1}{2}$
28(b)	Line perpendicular to the lines $\vec{r} = 2\hat{\imath} + \hat{\jmath} - 3\hat{k} + \lambda(\hat{\imath} + 2\hat{\jmath} + 5\hat{k})$ and $\vec{r} = 3\hat{\imath} + 3\hat{\jmath} - 7\hat{k} + \mu(3\hat{\imath} - 2\hat{\jmath} + 5\hat{k}).$		1
	has its $\vec{b} = \vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 2 & 5 \\ 3 & -2 & 5 \end{vmatrix} = 20\hat{\imath} + 10\hat{\jmath} - 8\hat{k}$		1
	$\therefore$ equation of line in vector form is $\vec{r} = -\hat{\imath} + 2\hat{\jmath} + 7\hat{k} + a(10\hat{\imath} + 5\hat{\jmath} - 4\hat{k})$		1
	And equation of line in cartesian form is $\dfrac{x+1}{10} = \dfrac{y-2}{5} = \dfrac{z-7}{-4}$		1
29.(a)	**Solution :** $\displaystyle\int \left\{ \dfrac{1}{\log_e x} - \dfrac{1}{(\log_e x)^2} \right\} dx$		
	$= \displaystyle\int \dfrac{dx}{\log_e x} - \int \dfrac{1}{(\log_e x)^2} dx = \dfrac{1}{\log_e x} \int dx - \int \left\{ \dfrac{d}{dx}\left(\dfrac{1}{\log_e x}\right) \int dx \right\} dx - \int \dfrac{1}{(\log_e x)^2} dx$		1
	$= \dfrac{x}{\log_e x} + \displaystyle\int \dfrac{1}{(\log_e x)^2} \dfrac{1}{x} . x . dx - \int \dfrac{1}{(\log_e x)^2} dx$		1
	$= \dfrac{x}{\log_e x} + \displaystyle\int \dfrac{1}{(\log_e x)^2} dx - \int \dfrac{dx}{(\log_e x)^2} = \dfrac{x}{\log_e x} + c \ ;$		1
	where 'c' is any arbitary constant of integration.		
OR **29.(b)**	$\displaystyle\int_0^1 x(1-x)^n dx$		
	$= \displaystyle\int_0^1 (1-x)\{1-(1-x)\}^n dx, \quad \left(as, \int_0^a f(x)dx = \int_0^a f(a-x)dx \right)$		1
	$= \displaystyle\int_0^1 x^n(1-x)dx$		$\dfrac{1}{2}$
	$= \displaystyle\int_0^1 x^n dx - \int_0^1 x^{n+1} dx$		
	$= \dfrac{1}{n+1}\left[x^{n+1}\right]_0^1 - \dfrac{1}{n+2}\left[x^{n+2}\right]_0^1$		
	$= \dfrac{1}{n+1} - \dfrac{1}{n+2}$		$\dfrac{1}{2}$
	$= \dfrac{1}{(n+1)(n+2)}.$		1

30.	Since the maximum value lies on the line segment CD	1
	Therefore, $Z_C = Z_D$	$\frac{1}{2}$
	$=> \quad 10m + 1500 = 1800 \quad\quad => m = 30$	$\frac{1}{2}$
	Thus objective function is $Z = 30(x + y)$	1
	Difference $= Z_C - Z_B = 1800 - 600 = 1200.$	
31.(a)	Since the event of raining today and not raining today are complementary events so if the probability that it rains today is 0.4 then the probability that it does not rain today is $1 - 0.4 = 0.6 \Rightarrow P_1 = 0.6$	
	If it rains today, the probability that it will rain tomorrow is 0.8 then the probability that it will not rain tomorrow is $1 - 0.8 = 0.2$.	$\frac{1}{2}$
	If it does not rain today, the probability that it will rain tomorrow is 0.7 then the probability that it will not rain tomorrow is $1 - 0.7 = 0.3$	
	(i) $\quad P_1 \times P_4 - P_2 \times P_3 = 0.6 \times 0.3 - 0.2 \times 0.7 = 0.04.$	1
	(ii) $\quad$ Let E_1 and E_2 be the events that it will rain today and it will not rain today respectively.	
	$\quad P(E_1) = 0.4 \ \& \ P(E_2) = 0.6$	$\frac{1}{2}$
	A be the event that it will rain tomorrow. $P\left(\frac{A}{E_1}\right) = 0.8 \ \& \ P\left(\frac{A}{E_2}\right) = 0.7$	$\frac{1}{2}$
	We have, $P(A) = P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right) = 0.4 \times 0.8 + 0.6 \times 0.7 = 0.74.$	$\frac{1}{2}$
	The probability of rain tomorrow is 0.74.	
OR 31.(b)	Given $P(X = r) \alpha \dfrac{1}{5^r}$	$\frac{1}{2}$

$P(X = r) = k \dfrac{1}{5^r}$, (where k is a non zero constant)

$P(r = 0) = k \cdot \dfrac{1}{5^0}$

$P(r = 1) = k \cdot \dfrac{1}{5^1}$

$P(r = 2) = k \cdot \dfrac{1}{5^2}$

$P(r = 3) = k \cdot \dfrac{1}{5^3}$

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We have, $P(X = 0) + P(X = 1) + P(X = 2) + \ldots\ldots\ldots = 1$

$\Rightarrow k \left(1 + \dfrac{1}{5} + \dfrac{1}{5^2} + \dfrac{1}{5^3} + \ldots\ldots\ldots\ldots \right) = 1$

$\dfrac{1}{2}$

$\Rightarrow k \left(\dfrac{1}{1 - \dfrac{1}{5}} \right) = 1 \Rightarrow k = \dfrac{4}{5}$

$\dfrac{1}{2}$

So, $P(X < 3) = P(X = 0) + P(X = 1) + P(X = 2)$

$= \dfrac{4}{5} \left(1 + \dfrac{1}{5} + \dfrac{1}{5^2} \right) = \dfrac{4}{5} \left(\dfrac{25 + 5 + 1}{25} \right) = \dfrac{124}{125}$.

1

Section –D

[This section comprises of solution of long answer type questions (LA) of 5 marks each]

32.	Area Bounded by the given curve (ellipse) is $A = 4 \int_0^2 \sqrt{1 - \dfrac{x^2}{4}}\, dx$	2
	$A = 4 \int_0^2 \left(\dfrac{1}{2} \sqrt{4 - x^2} \right) dx$	1
	$A = 2 \left[\dfrac{x\sqrt{4 - x^2}}{2} + \dfrac{4}{2} \sin^{-1} \dfrac{x}{2} \right]_0^2$	1
	$A = 2\pi$ sq. units.	1

33.	$y = ax^2 + bx + c$	
	$15 = 4a + 2b + c$	
	$25 = 16a + 4b + c$	1
	$15 = 196a + 14b + c$	
	The set of equations can be represented in the matrix form as $AX = B$,	
	where $A = \begin{bmatrix} 4 & 2 & 1 \\ 16 & 4 & 1 \\ 196 & 14 & 1 \end{bmatrix}$, $X = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$ and $B = \begin{bmatrix} 15 \\ 25 \\ 15 \end{bmatrix} \Rightarrow \begin{bmatrix} 4 & 2 & 1 \\ 16 & 4 & 1 \\ 196 & 14 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 15 \\ 25 \\ 15 \end{bmatrix}$.	$\dfrac{1}{2}$
	$|A| = 4(4 - 14) - 2(16 - 196) + (224 - 784) = -40 + 360 - 560 = -240 \neq 0$. Hence A^{-1} exists.	$\dfrac{1}{2}$

Now, $adj(A) = \begin{bmatrix} -10 & 180 & -560 \\ 12 & -192 & 336 \\ -2 & 12 & -16 \end{bmatrix}^T = \begin{bmatrix} -10 & 12 & -2 \\ 180 & -192 & 12 \\ -560 & 336 & -16 \end{bmatrix}$		1
$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = -\frac{1}{240}\begin{bmatrix} -1 & 12 & -2 \\ 180 & -192 & 12 \\ -560 & 336 & -16 \end{bmatrix}\begin{bmatrix} 15 \\ 25 \\ 15 \end{bmatrix} = -\frac{5}{240}\begin{bmatrix} -10 & 12 & -2 \\ 180 & -192 & 12 \\ -560 & 336 & -16 \end{bmatrix}\begin{bmatrix} 3 \\ 5 \\ 3 \end{bmatrix} = -\frac{5}{240}\begin{bmatrix} 24 \\ -384 \\ -48 \end{bmatrix}$		1
$\therefore a = -\frac{1}{2}, b = 8, c = 1$		$\frac{1}{2}$
So, the equation becomes $y = -\frac{1}{2}x^2 + 8x + 1$		$\frac{1}{2}$

34.(a)	We have, $f(x) = |x|^3, \begin{cases} x^3, \text{if } x \geq 0 \\ (-x)^3 = -x^3, \text{if } x < 0 \end{cases}$	$\frac{1}{2}$
	Now, $(LHD at x = 0) = \lim_{x \to 0^-} \frac{f(x) - f(0)}{x - 0} = \lim_{x \to 0^-}\left(\frac{-x^3 - 0}{x}\right) = \lim_{x \to 0^-}(-x^2) = 0$	$\frac{1}{2}$
	$(RHD at x = 0) \lim_{x \to 0^+}\frac{f(x) - f(0)}{x - 0} = \lim_{x \to 0^+}\left(\frac{x^3 - 0}{x}\right) = \lim_{x \to 0}(-x^2) = 0$	$\frac{1}{2}$ $\frac{1}{2}$
	$\therefore (LHD of f(x) at x = 0) = (RHD of f(x) at x = 0)$	1
	So, $f(x)$ is differentiable at $x = 0$ and the derivative of $f(x)$ is given by	
	$f'(x) = \begin{cases} 3x^2, \text{if } x \geq 0 \\ -3x^2, \text{if } x < 0 \end{cases}$	$\frac{1}{2}$
	Now, $(LHD of f'(x) at x = 0) = \lim_{x \to 0^-}\frac{f'(x) - f'(0)}{x - 0} = \lim_{x \to 0^-}\left(\frac{-3x^2 - 0}{x}\right) = \lim_{x \to 0^-}(-3x) = 0$	$\frac{1}{2}$
	$(RHD of f'(x) at x = 0) = \lim_{x \to 0^+}\frac{f'(x) - f'(0)}{x - 0} = \lim_{x \to 0^+}\left(\frac{3x^2 - 0}{x - 0}\right) = \lim_{x \to 0^+}(3x) = 0$	$\frac{1}{2}$
	$\therefore (LHD of f'(x) at x = 0) = (RHD of f'(x) at x = 0)$	
	So, $f'(x)$ is differentiable at $x = 0.$	$\frac{1}{2}$
	Hence, $f''(x) = \begin{cases} 6x, \text{if } x \geq 0 \\ -6x, \text{if } x < 0. \end{cases}$	$\frac{1}{2}$

OR 34.(b)	Given relation is $(x - a)^2 + (y - b)^2 = c^2, c > 0.$	
	Let $x - a = c\cos\theta$ and $y - b = c\sin\theta.$	$\frac{1}{2}$
	Therefore, $\frac{dx}{d\theta} = -c\sin\theta$ And $\frac{dy}{d\theta} = c\cos\theta$	$\frac{1}{2}$
	$\therefore \frac{dy}{dx} = -\cot\theta$	1
	Differentiate both sides with respect to θ, we get $\frac{d}{d\theta}\left(\frac{dy}{dx}\right) = \frac{d}{d\theta}(-\cot\theta)$	$\frac{1}{2}$

Or, $\frac{d}{dx}\left(\frac{dy}{dx}\right)\frac{dx}{d\theta} = cosec^2\theta$

$\frac{1}{2}$

Or, $\frac{d^2y}{dx^2}(-c\sin\theta) = cosec^2\theta$

$\frac{1}{2}$
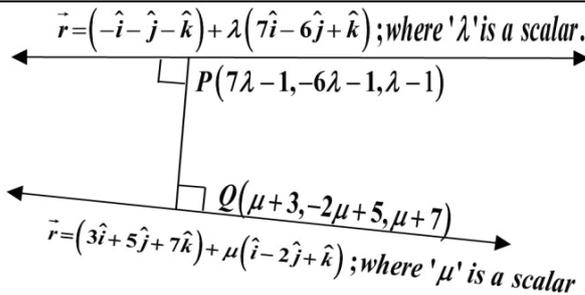
$\frac{d^2y}{dx^2} = -\frac{cosec^3\theta}{c}$

1

$\therefore \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}} = \frac{c[1+cot^2\theta]^{\frac{3}{2}}}{-cosec^3\theta} = \frac{-c(cosec^2\theta)^{\frac{3}{2}}}{cosec^3\theta} = -c,$

$\frac{1}{2}$

Which is constant and is independent of a and b.

35.(a)	

$\vec{r} = \left(-\hat{i}-\hat{j}-\hat{k}\right)+\lambda\left(7\hat{i}-6\hat{j}+\hat{k}\right); \text{where '}\lambda\text{'is a scalar.}$

$P(7\lambda-1,-6\lambda-1,\lambda-1)$

$Q(\mu+3,-2\mu+5,\mu+7)$

$\vec{r} = \left(3\hat{i}+5\hat{j}+7\hat{k}\right)+\mu\left(\hat{i}-2\hat{j}+\hat{k}\right); \text{where '}\mu\text{' is a scalar}$

Given that equation of lines are

$\vec{r} = \left(-\hat{i}-\hat{j}-\hat{k}\right)+\lambda\left(7\hat{i}-6\hat{j}+\hat{k}\right)\ldots\ldots\ldots\ldots(i)$ and

$\vec{r} = \left(3\hat{i}+5\hat{j}+7\hat{k}\right)+\mu\left(\hat{i}-2\hat{j}+\hat{k}\right)\ldots\ldots\ldots\ldots(ii)$

The given lines are non-parallel lines as vectors $7\hat{i}-6\hat{j}+\hat{k}$ and $\hat{i}-2\hat{j}+\hat{k}$ are not parallel. There is a

unique line segment PQ (P lying on line (i) and Q on the other line (ii)), which is at right angles

to both the lines PQ is the shortest distance between the lines.

Hence, the shortest possible distance between the lines $= PQ$.

Let the position vector of the point P lying on the line $\vec{r} = \left(-\hat{i}-\hat{j}-\hat{k}\right)+\lambda\left(7\hat{i}-6\hat{j}+\hat{k}\right)$ where'λ'

is a scalar, is $(7\lambda-1)\hat{i}-(6\lambda+1)\hat{j}+(\lambda-1)\hat{k}$, for some λ and the position vector of the point Q

lying on the line $\vec{r} = \left(3\hat{i}+5\hat{j}+7\hat{k}\right)+\mu\left(\hat{i}-2\hat{j}+\hat{k}\right)$ where 'μ'is a scalar, is

$(\mu+3)\hat{i}+(-2\mu+5)\hat{j}+(\mu+7)\hat{k}$, for some μ. Now, the vector

$\overrightarrow{PQ} = \overrightarrow{OQ}-\overrightarrow{OP} = (\mu+3-7\lambda+1)\hat{i}+(-2\mu+5+6\lambda+1)\hat{j}+(\mu+7-\lambda+1)\hat{k}$

$i.e., \overrightarrow{PQ} = (\mu-7\lambda+4)\hat{i}+(-2\mu+6\lambda+6)\hat{j}+(\mu-\lambda+8)\hat{k}$; (where '$O$' is the origin), is

perpendicular to both the lines, so the vector $\overrightarrow{PQ}$ is perpendicular to both the vectors $7\hat{i}-6\hat{j}+\hat{k}$ and

$\hat{i}-2\hat{j}+\hat{k}$.

$\Rightarrow (\mu-7\lambda+4).7 + (-2\mu+6\lambda+6).(-6) + (\mu-\lambda+8).1 = 0$

$\frac{1}{2}$

$\frac{1}{2}$

1

$\&(\mu - 7\lambda + 4).1 + (-2\mu + 6\lambda + 6).(-2) + (\mu - \lambda + 8).1 = 0$

$\Rightarrow \mathbf{20\mu - 86\lambda = 0} \implies \mathbf{10\mu - 43\lambda = 0}\ \& 6\mu - 20\lambda = 0 \Rightarrow 3\mu - 10\lambda = 0$ | **1**
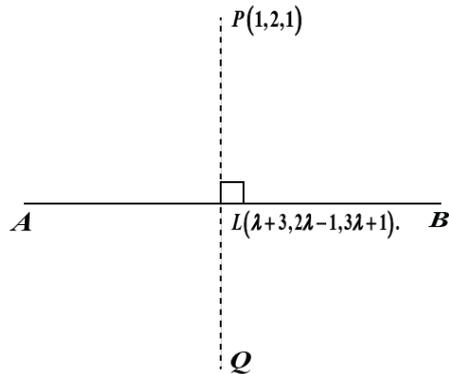
On solving the above equations, we get $\mu = \lambda = 0$ | $\dfrac{1}{2}$

So, the position vector of the points **P** and **Q** are $-\hat{\imath} - \hat{\jmath} - \hat{k}$ and $3\hat{\imath} + 5\hat{\jmath} + 7\hat{k}$ respectively.

$\overrightarrow{PQ} = 4\hat{\imath} + 6\hat{\jmath} + 8\hat{k}$ and | $\dfrac{1}{2}$

$\left|\overrightarrow{PQ}\right| = \sqrt{4^2 + 6^2 + 8^2} = \sqrt{116} = 2\sqrt{29}\ units.$ | **1**

OR

35.(b)

Let $P(1, 2, 1)$ be the given point and L be the foot of the perpendicular from P to the given line AB (*as shown in the figure above*).

Let's put $\dfrac{x-3}{1} = \dfrac{y+1}{2} = \dfrac{z-1}{3} = \lambda.$ Then, $x = \lambda + 3, y = 2\lambda - 1, z = 3\lambda + 1$ | $\dfrac{1}{2}$

Let the coordinates of the point L be $(\lambda + 3, 2\lambda - 1, 3\lambda + 1).$

So, direction ratios of PL are $(\lambda + 3 - 1, 2\lambda - 1 - 2, 3\lambda + 1 - 1)\ i.e., (\lambda + 2, 2\lambda - 3, 3\lambda)$ | $\dfrac{1}{2}$

Direction ratios of the given line are **1, 2 and 3,** which is perpendicular to PL. Therefore, we have,

$$(\lambda + 2).1 + (2\lambda - 3).2 + 3\lambda.3 = 0 \Rightarrow 14\lambda = 4 \Rightarrow \lambda = \dfrac{2}{7}$$ | $\dfrac{1}{2}$

Then, $\lambda + 3 = \dfrac{2}{7} + 3 = \dfrac{23}{7};\ 2\lambda - 1 = 2\left(\dfrac{2}{7}\right) - 1 = -\dfrac{3}{7};\ 3\lambda + 1 = 3\left(\dfrac{2}{7}\right) + 1 = \dfrac{13}{7}$ | $\dfrac{1}{2}$

Therefore, coordinates of the point L are $\left(\dfrac{23}{7}, -\dfrac{3}{7}, \dfrac{13}{7}\right).$

Let $Q(x_1, y_1, z_1)$ be the image of $P(1, 2, 1)$ with respect to the given line. Then, L is the mid-point of $PQ.$ | **1**

Therefore, $\dfrac{1+x_1}{2} = \dfrac{23}{7}, \dfrac{2+y_1}{2} = -\dfrac{3}{7}, \dfrac{1+z_1}{2} = \dfrac{13}{7} \Rightarrow x_1 = \dfrac{39}{7}, y_1 = -\dfrac{20}{7}, z_1 = \dfrac{19}{7}$

Hence, the image of the point $P(1,2,1)$ with respect to the given line $Q\left(\dfrac{39}{7}, -\dfrac{20}{7}, \dfrac{19}{7}\right).$ | **1**

The equation of the line joining $P(1,2,1)$ and $Q\left(\dfrac{39}{7}, -\dfrac{20}{7}, \dfrac{19}{7}\right)$ is

	$\dfrac{x-1}{32/7} = \dfrac{y-2}{-34/7} = \dfrac{z-1}{12/7} \Rightarrow \dfrac{x-1}{16} = \dfrac{y-2}{-17} = \dfrac{z-1}{6}.$	**1**

[This section comprises solution of 3 case- study/passage based questions of 4 marks each with two sub parts. Solution of the first two case study questions have three sub parts (i),(ii),(iii) of marks 1,1,2 respectively. Solution of the third case study question has two sub parts of 2 marks each.)

36.	(i) $V = (40 - 2x)(25 - 2x)x \, cm^3$	**1**
	(ii) $\dfrac{dV}{dx} = 4(3x - 50)(x - 5)$	**1**
	(iii)For extreme values $\dfrac{dV}{dx} = 4(3x - 50)(x - 5) = 0$	**½**
	$\Rightarrow x = \dfrac{50}{3}$ or $x = 5$	**½**
	$\dfrac{d^2V}{dx^2} = 24x - 260$	**½**
	$\therefore \dfrac{d^2V}{dx^2}$ at $x = 5$ is $-140 < 0$	**½**
	$\therefore V$ is max $when \; x = 5$	
	(iii) **OR**	
	For extreme values $\dfrac{dV}{dx} = 4(3x^2 - 65x + 250)$	**½**
	$\dfrac{d^2V}{dx^2} = 4(6x - 65)$	**½**
	$\dfrac{dV}{dx}$ at $x = \dfrac{65}{6}$ exists and $\dfrac{d^2V}{dx^2}$ at $x = \dfrac{65}{6}$ is 0.	
	$\dfrac{d^2V}{dx^2}$ at $x = \left(\dfrac{65}{6}\right)^{-}$ is negative and $\dfrac{d^2V}{dx^2}$ at $x = \left(\dfrac{65}{6}\right)^{+}$ is positive	**½**
	$\therefore x = \dfrac{65}{6}$ is a point of inflection.	**½**
37.	(i) **Number of relations is equal to the number of subsets of the set $B \times G = 2^{n(B \times G)}$** $= 2^{n(B) \times n(G)} = 2^{3 \times 2} = 2^6$ ($Where n(A)$ $denotes \; the \; number \; of \; the \; elements \; in \; the \; finite \; set \; A$)	**1**
	(ii) **Smallest Equivalence relation on G is $\left\{(g_1, g_1), (g_2, g_2)\right\}$**	**1**
	(iii) (A) reflexive but not symmetric $=$ $\{(b_1, b_2), (b_2, b_1), (b_1, b_1), (b_2, b_2), (b_3, b_3), (b_2, b_3)\}$. **So the minimum number of elements to be added are** $(b_1, b_1), (b_2, b_2), (b_3, b_3), (b_2, b_3)$	**1**

{Note : it can be any one of the pair from, (b_3, b_2), (b_1, b_3), (b_3, b_1) in place of (b_2, b_3) also}

(B) reflexive and symmetric but not transitive =

$\{(b_1, b_2), (b_2, b_1), (b_1, b_1), (b_2, b_2), (b_3, b_3), (b_2, b_3), (b_3, b_2)\}$.

So the minimum number of elements to be added are

$(b_1, b_1), (b_2, b_2), (b_3, b_3), (b_2, b_3), (b_3, b_2)$

1

OR (iii) (b) One-one and onto function

$x^2 = 4y$. let $y = f(x) = \dfrac{x^2}{4}$

Let $x_1, x_2 \in [0, 20\sqrt{2}]$ such that $f(x_1) = f(x_2) \Rightarrow \dfrac{x_1^2}{4} = \dfrac{x_1^2}{4}$

$\Rightarrow x_1^2 = x_2^2 \Rightarrow (x_1 - x_2)(x_1 + x_2) = 0 \Rightarrow x_1 = x_2$ as $x_1, x_2 \in [0, 20\sqrt{2}]$

$\therefore f$ is one-one function

Now, $0 \le y \le 200$ hence the value of y is non-negative

and $f(2\sqrt{y}) = y$

$\therefore$ for any arbitrary $y \in [0, 200]$, the pre-image of y exists in $[0, 20\sqrt{2}]$

hence f is onto function.

1

1

1

38.

Let E_1 be the event that one parrot and one owl flew from cage –I

E_2 be the event that two parrots flew from Cage-I

A be the event that the owl is still in cage-I

(i) Total ways for A to happen

From cage I 1 parrot and 1 owl flew and then from Cage-II 1 parrot and 1 owl flew back + From cage I 1 parrot and 1 owl flew and then from Cage-II 2 parrots flew back + From cage I 2 parrots flew and then from Cage-II 2 parrots came back.

$= (5_{C_1} \times 1_{C_1})(7_{C_1} \times 1_{C_1}) + (5_{C_1} \times 1_{C_1})(7_{C_2}) + (5_{C_2})(8_{C_2})$

$\dfrac{1}{2}$

Probability that the owl is still in cage –I = $P(E_1 \cap A) + P(E_2 \cap A)$

$\dfrac{(5_{C_1} \times 1_{C_1})(7_{C_1} \times 1_{C_1}) + (5_{C_2})(8_{C_2})}{(5_{C_1} \times 1_{C_1})(7_{C_1} \times 1_{C_1}) + (5_{C_1} \times 1_{C_1})(7_{C_2}) + (5_{C_2})(8_{C_2})}$

1

$= \dfrac{35 + 280}{35 + 105 + 280} = \dfrac{315}{420} = \dfrac{3}{4}$

$\dfrac{1}{2}$

(i) The probability that one parrot and the owl flew from Cage-I to Cage-II given that the owl is still in cage-I is $P\left(E_1/A\right)$

$\dfrac{1}{2}$

$$P\left(E_1/A\right) = \frac{P(E_1 \cap A)}{P(E_1 \cap A) + P(E_2 \cap A)} \text{ (by Baye's Theorem)}$$

$$= \frac{\frac{35}{420}}{\frac{315}{420}} = \frac{1}{9}$$

$\frac{1}{2}$

1