CLASS- XII

SUBJECT: Applied Mathematics (241)

Time: 3 Hours.   **Maximum Marks: 80**

General Instructions:

Read the following instructions very carefully and strictly follow them:

(i)   This Question paper contains **38** questions. **All** questions are **compulsory.**

(ii)   This Question paper is divided into **five** Sections - **A, B, C, D** and **E.**

(iii) In **Section A**, Questions no. **1** to **18** are **multiple choice questions (MCQs)** and Questions no. **19** and **20** are **Assertion-Reason based** questions of **1 mark each**.

(iv)   In **Section B,** Questions no. **21** to **25** are **Very Short Answer (VSA)-type** questions, carrying **2 marks each**.

(v)   In **Section C,** Questions no. **26** to **31** are **Short Answer (SA)-type** questions, carrying **3 marks each**.

(vi)   In **Section D,** Questions no. **32** to **35** are **Long Answer (LA)-type** questions, carrying **5 marks each**.

(vii) In **Section E,** Questions no. **36** to **38** are **case study-based questions** carrying **4 marks each.**

(viii) There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 2 questions in Section C, 2 questions in Section D and one sub-part each in 2 questions of Section E.

(ix)   Use of calculators is **not** allowed.

SECTION-A $\left[1 \times 20 = 20\right]$

(***This section comprises of multiple-choice questions (MCQs) of 1 mark each***)

Select the correct option (Question 1 - Question 18):

Q.1. The area (in sq units) bounded by the curve $y = \sqrt{x}$, the x–axis, $x = 1$ and $x = 4$ is

(A) $\frac{11}{3}$ (B) $\frac{1}{4}$ (C) $\frac{14}{3}$ (D) $\frac{13}{3}$

**Q.2.** Sampling which provides for a known non-zero equal chance of selection is

(A) Systematic sampling   (B) Convenience sampling

(C) Quota sampling   (D) Purposive sampling

Q.3. Let the cost function for a manufacturer is given by $C(x) = \frac{x^3}{3} - x^2 + 2x$ (In rupees)

Which of the following statement is correct based on the above information?

(A) The marginal cost decreases from 0 to 1 and then increases onwards.

(B) The marginal cost increases from 0 to 1 and then decreases onwards.

(C) Marginal cost decreases as production level increases from zero.

(D) Marginal cost increases as production level increases from zero.

Q.4. The absolute minimum value of the function $f(x) = 4x - \dfrac{1}{2}x^2$ in the interval $\left[-2, \dfrac{9}{2}\right]$ is:

(A) -8 (B) -9 (C) -10 (D) -16

Q.5. For the purpose of $t-$ test of significance, a random sample of size $(n)\,2025$ is drawn from a normal population, then the degree of freedom (v) is

(A) 2025^{2025} (B) 2024^{2025} (C) 2025 (D) 2024

Q.6. The constraints of a linear programming problem along with their graphs is shown below:

$x + 2y \geq 3, \ x \geq 10, \ y \geq 0$

Which of the following inequality may be removed so that the feasible region remains the same in above graph?

(A) $x + 2y \geq 3$

(B) $x \geq 10$

(C) $y \geq 0$

(D) $x \geq 0$

Q.7. A player rolls one fair die. If the die shows an odd number, the player wins the value that appears on the die, else loses half the value that appears on it. The expected gain of the player is

(A) $-\dfrac{1}{2}$ (B) 0 (C) $\dfrac{1}{2}$ (D) 1

Q.8. The original cost of a machine is ₹ 1200000 and the scarp value of the machine after a useful life of **3 years is** ₹ 300000, then the book value of the machine at the end **2 years** is

(A) ₹ 100000   (B) ₹ 250000   (C) ₹ 600000   (D) ₹ 800000

Q.9. A fish jumps out of the water surface and follows the parabolic path $y = 6x - x^2 - 8;\ 2 \le x \le 4$.

The fish reaches the highest height in its path at $(3, 1)$. The slope of the path of the fish at

$(3, 1)$ is

(A) **0**   (B) **1**   (C) **2**   (D) **3**

Q.10. In a large consignment of electric bulbs **5%** of a batch of batteries are defective. A random sample of **80** is taken for inspection with replacement. Then the Variance of the number of defectives in the sample, is

(A) $\dfrac{18}{5}$ (B) $\dfrac{19}{5}$ (C) **4.555**   (D) **8**

Q.11. If it is currently $6:00$ pm in 12 hours clock then what will be the time after 375 hours?

(A) 6 am   (B) 6 pm   (C) 9 am   (D) 9 pm

Q.12. The values of $\frac{1}{x}$ for the given values of $x \in (-1, 3) - \{0\}$ is

(A) $\left(-1, \frac{1}{3}\right) \cup (3, \infty)$ (B) $(-\infty, -1) \cup \left(\frac{1}{3}, \infty\right)$ (C) $\left(-\frac{1}{3}, 1\right)$ (D) $\left(-\frac{1}{3}, -1\right)$

Q.13. The component of a time series attached to long term variations is termed as

(A) Seasonal variations   (B) Irregular variations

(C) Secular trend variations   (D) Cyclic variations

Q.14. The present value of a sequence of payments of **₹ 800** made at the end of every **6 month** and continuing forever. If money is worth **4%** per annum compounded semi-annually, then the present value of the sequence is:

(A) ₹ 20000   (B) ₹ 40000   (C) ₹ 60000   (D) ₹ 80000

Q.15. Shown below is a curve.
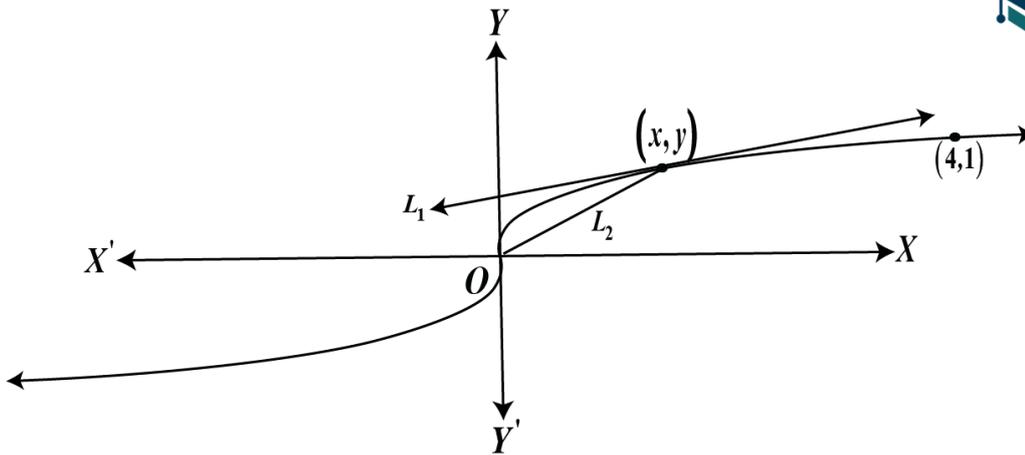
L_1 is the tangent to any point (x, y) on the curve.

L_2 is the line that connects the point (x, y) to the origin.

The slope of L_1 is one third of the slope of L_2.

Then the differential equation, using the given conditions is:

(A) $\frac{dy}{dx} = \frac{y}{3x}$ (B) $\frac{dy}{dx} = \frac{y}{x}$ (C) $\frac{dy}{dx} = \frac{x}{3y}$ (D) $\frac{dy}{dx} = \frac{3y}{x}$

Q.16. For a 3×3 matrix if adj $A = 2A^{-1}$, find $|3AA^T|$

(A) 108   (B) 12   (C) 54   (D) 8

Q.17. For two matrices $P = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}$ & $Q^T = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$; (where Q^T is the transpose of the matrix Q)

, $P - Q$ is:

(A) $\begin{bmatrix} 2 & 3 \\ -3 & 0 \\ 0 & -3 \end{bmatrix}$ (B) $\begin{bmatrix} 4 & 3 \\ -3 & 0 \\ -1 & -2 \end{bmatrix}$ (C) $\begin{bmatrix} 4 & 3 \\ 0 & -3 \\ -1 & -2 \end{bmatrix}$ (D) $\begin{bmatrix} 2 & 3 \\ 0 & -3 \\ 0 & -3 \end{bmatrix}$

Q.18. The order and degree of a differential equation $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^4 + x^{\frac{1}{5}} = 0$; respectively, are

(A) 2 and 4   (B) 2 and 1
(C) 2 and 3   (D) 3 and 3

ASSERTION-REASON BASED QUESTIONS

(Questions number 19 and 20 are Assertion and Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below.)

$$[1 \times 2 = 2]$$

(A) Both (A) and (R) are true and (R) is the correct explanation of (A).

(B) Both (A) and (R) are true but (R) is not the correct explanation of (A).

(C) (A) is true but (R) is false.

(D) (A) is false but (R) is true.

Q.19. Assertion (A): The effective rate of interest equivalent to a nominal rate of 6% when compounded continuously is equal to $e^{0.06} - 1 = 6.18\%$.

Reason (R): The relation between effective rate $\left(r_{eff}\right)$ of interest and nominal rate (r) of interest: $r_{eff} = e^r - 1;$ where $'e'$ - Euler's number (approximate value is 2.71828), when compounded continuously.

Q.20. Assertion(A): $A = \left[a_{ij}\right] = \begin{cases} m; i = j \\ 0; i \neq j \end{cases}$

where m is a scalar, is an identity matrix if $m = 1$

Reason (R): Every identity matrix is not a scalar matrix

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SECTION B $\qquad \left[2 \times 5 = 10\right]$

(This section comprises of 5 very short answer (VSA) type questions of 2 marks each.)

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Q.21. (a) In what ratio water must be added in milk costing ₹ 60 per litre, so that the resulting mixture would be of worth ₹ 50 per litre?

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OR

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Q.21. (b) A pump can fill a tank with water in 2 hours. Because of leakage, it took $\frac{7}{3}$ hrs to fill the tank. How much time will it take for the leakage to drain all the water in the full tank?

Q.22. In a 200 m race, A can give a start of 18 m to B and a start of 31 m to C. In a race of 350 m, how much start can B give to C?

Q.23. A boat takes thrice as long to go upstream to a point as to return downstream to the starting point. If the speed of the stream is $5 km/h$, find the speed of the boat in still water.

Q.24. (a) The incidence of occupational disease in an industry is such that the workers have a **20%** chance of suffering from it. What is the probability that out of six workers **4 or more** will catch the disease?

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OR

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Q.24. (b) The lifetime of an item produced by a machine has a normal distribution with **mean 12 months** and **standard deviation** of 2 months. Find the probability of an item produced by this machine will last

(i)   less than **7** months

(ii)   between **7** and **14** months.

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(Given $P\left(Z < \frac{5}{2}\right) = 0.9938$ **and** $P(Z < 1) = 0.8413$ **)**

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Q.25. If $A = \begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$, then find the value of α (if exists) for which $A^2 = B$.

$$\left[3 \times 6 = 18\right]$$

(This section comprises of 6 short answer (SA) type questions of 3 marks each.)

Q.26. Find the remainder when 5^{61} is divided by 7.

Q.27. (a) Two batches of the same product are tested for their mean life. Assuming that, the lives of the product follow a normal distribution with an unknown variance; test the hypothesis that the mean life is the same for both the branches, given the following information:

Batch	Sample Size	Mean life (in hours)	Standard Deviation (in hours)
Batch I	10	750	12
Batch II	8	820	14

$$\left[\text{Given } \sqrt{4.4444} = 2.1081 \text{ and } t_{16}\left(0.05\right) = 2.120\right]$$

OR

Q.27. (b) The manufacturer of electrical items makes bulbs and claims that these bulbs have a mean life of 25 months. The life in months of a random sample of 6 such bulbs are given to be 24, 26, 30, 20, 20 and 18. Test the validity of the manufacturer's claim at 1% level of significance. [Given $t_5(0.01) = 4.032$]

Q.28. A traffic engineer records the number of bicycle riders that use a particular cycle track. He records that an average of 3.2 bicycle riders use the cycle track every hour. Given that the number of bicycles that use the cycle track follow a Poisson distribution, what is the probability that 2 or less bicycle riders will use the cycle track within an hour? Also find the mean expectation and variance for the random variable. (Given $e^{-3.2} = 0.041$)

Q.29. Mr Rohit invested ₹ **5000** in a fund at the beginning of year **2021** and by the end of year **2021** his investment was worth ₹ **9000**. Next year market crashed and he lost ₹ **3000** and ending up with ₹ **6000** at the end of year **2022**. Next year i.e. **2023** he gained ₹ **4500** and ending up with ₹ **10500** at the end of the year. Find **CAGR** (Compounded Annual Growth Rate) of his investment. (Use $\left(2.1\right)^{1/3} = 1.2805$)

Q.30. A small firm manufactures necklaces and bracelets. The total number of necklaces and bracelets that it can handle per day is at most **25**. It takes one hour to make a bracelet and half

an hour to make a necklace. The maximum number of hours available per day is 14. If the profit on a necklace is ₹ **100** and that on a bracelet is ₹ **300**, formulate an **L.P.P**. for finding how many of each should be produced daily to maximize the profit? It is being given that at least one of each must be produced.

(Note: No need to find the feasible region and optimal solution)

Q.31.(a) An octagonal prism is a three-dimensional polyhedron bounded by two octagonal bases and eight rectangular side faces. It has **24** edges and **16** vertices.
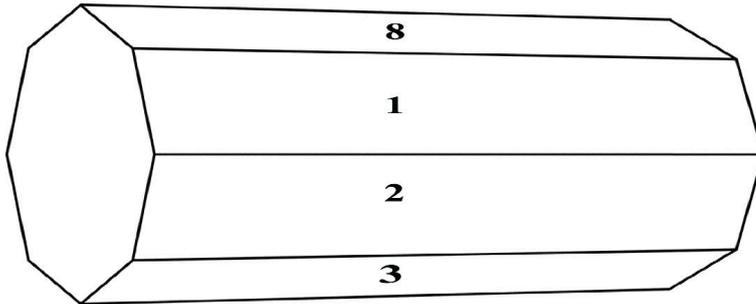
The prism is rolled along the rectangular faces and number on the bottom face (touching the ground) is noted. Let X denotes the number obtained on the bottom face and the following table gives the probability distribution of X.

$X:$	1	2	3	4	5	6	7	8
$P(X):$	p	$2p$	$2p$	p	$2p$	p^2	$2p^2$	$7p^2 + p$

   On the above context, answer the following questions.

   (i)   Find the value of p.

   (ii)   Find the mean, $E(X)$.

                   OR

Q.31.(b) If the probability of success in a single trial is **0.01**, how many minimum number of Bernoulli trials must be performed in order that the probability of at least one success is $\dfrac{1}{2}$ or more?

$\left(\text{Use } \log_{10} 2 = 0.3010 \text{ and } \log_{10} 99 = 1.9956 \right)$

SECTION D $[5 \times 4 = 20]$
(This section comprises of 4 long answer (LA) type questions of 5 marks each)

Q.32. (a) Fit a straight-line trend by using the method of least squares for the following data and calculate the trend values.

Year	Production (in tonnes)
1962	2
1963	4
1964	3
1965	4
1966	4
1967	2
1968	4
1969	9
1970	7
1971	10
1972	8

OR

Q.32. (b) The quarterly profits of a small-scale industry (₹ in thousands) are as follows.

Year	Quarter 1	Quarter 2	Quarter 3	Quarter 4
2020	39	47	20	56
2021	68	59	66	72
2022	88	60	60	67

Calculate **4-quarterly** moving averages.

Q.33. (a) An owl was sitting at $(0, k); k > 0$. Then it starts flying along the path whose equation is given by $y = ax^2 + bx + c,$ where $a \in \mathbb{R} - \{0\}$, $b, c \in \mathbb{R}$. It passes through the points $(1, 2), (2, 1)$ and $(4, 5)$. Using **Cramer's Rule**, find the values of a, b, c and hence k

OR

Q.33. (b) A toy rocket is fired, from a platform, vertically into the air, its height above the ground after t seconds is given by $s(t) = at^2 + bt + c$, where $a, b, c \in \mathbb{R}; a \neq 0$ and $s(t)$ is measured in

metres. After **10** second, the rocket is **16 m** above the ground; after **20** seconds, **22 m**, after **30** seconds, **25 m**.

(i) Write down a system of three linear equations in terms of a, b and c.

(ii) Hence find the values of a, b and c, using matrix method.

Q.34. Supply and demand curves of a tyre manufacturer company is given below:
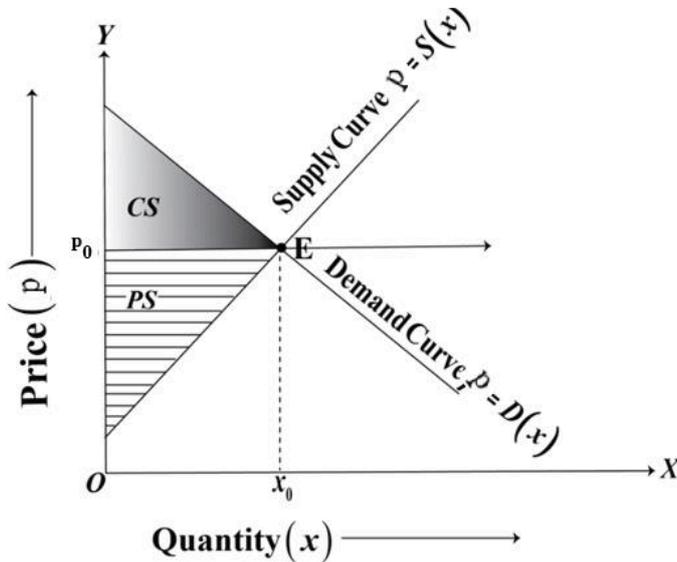
The above graph showing the demand and supply curves of a tyre manufacturer company which are linear. 'ABC' tyre manufacturer sold **25** units every month when the price of a tyre was ₹ **20000** per units and 'ABC' tyre manufacturer sold **125** units every month when the price dropped to ₹ **15000** per unit. When the price was ₹ **25000** per unit, **180** tyres were available per month for sale and when the price was only ₹ **15000** per unit, **80** tyres remained. Find the demand function. Also find the consumer surplus if the supply function is given to be $S(x) = 100x + 7000$

Q.35. In **4** years, a mobile costing ₹ **36,000** will have a salvage value of ₹ **7200**.

The following graph shows the depreciation of a mobile's value over 4 years.
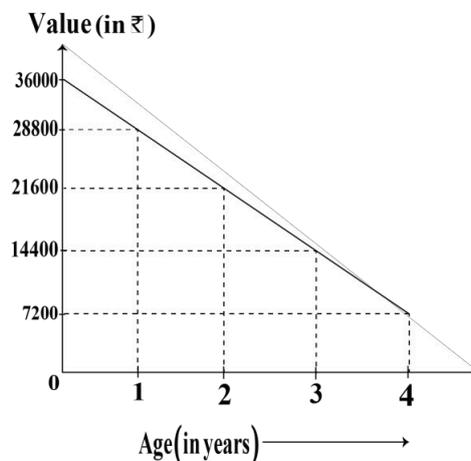
A new mobile at that time (i.e., after **4 years**) is expected to cost for ₹ **55,200**. In order to provide funds for the difference between the replacement cost and the salvage cost, a sinking

fund is set up into which equal payments are placed at the end of each year. If the fund earns interest at the rate 7% compounded annually, how much should each payment be? Also find the amount of Annual Depreciation of the mobile's value over **4** years and find the rate of depreciation (under straight line method). Use $(1.07)^4 = 1.3107$.

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SECTION- E $\qquad\qquad [4 \times 3 = 12]$

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(This section comprises of 3 case-study/passage-based questions of 4 marks each with sub parts. The first two case study questions have three sub parts (i), (ii), (iii) of marks 1, 1, 2 respectively. The third case study question has two sub parts of 2 marks each)

Case Study-1

Q.36. A student Shivam is running on a playground along the curve given by $y = x^2 + 7$. Another student Manita standing at point $(3, 7)$ on playground wants to hit Shivam by paper ball when Shivam is nearest to Manita.

Based on above information, answer the following questions:

(i) Let at any instant while running along the curve $y = x^2 + 7$, Shivam's position be (x, y). Find the expression for the distance (D) between Shivam and Manita in terms of $'x'$. [**1**]

(ii) Find the critical point(s) of the distance function. [**1**]

(iii) (a) What is the distance between Shivam and Manita when they are at least distance from each other. [**2**]

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OR

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(iii) (b) Find the position of Shivam, when he is closest to Manita. [**2**]

Case Study-2

Q.37. EQUATED MONTHLY INSTALMENTS (EMI): -

Each instalment can be considered as consisting of two parts:

(i) Interest on the outstanding loan $\qquad\qquad$ (ii) Repayment of part of the loan.

Methods of calculation of EMI or Instalment: -

EMI or Installment can be calculated by two methods:

1. Flat Rate Method

2. Reducing-balance method or Amortization of Loan

Rajesh purchased a house from a company for ₹ **2500000** and made a down payment of ₹ **500000** He repays the balance in **25 years** by monthly instalments at the rate of **9%** per annum compounded monthly. $\left(\text{Given}\,(1.0075)^{-300} = 0.1062\right)$

Based on the above information, answer the following questions:

(i) Find the number of payments and find the rate of interest per month. [**1**]

(ii) (a) What are the monthly payments of instalments using **reducing balance method**?
[**2**]

OR

(ii) (b) What are the monthly payments of instalments using **flat rate method**? [**2**]

(iii) What is the total interest payment made in the process applied to calculate **EMI** in the above part $(37(ii))$? [**1**]

Case Study- 3

Q.38. A company has two factories located at **P** and **Q** and has three depots situated at **A, B** and **C**. The weekly requirement of the depots at **A, B** and **C** is respectively **5, 5** and **4** units, while the production capacity of the factories **P** and. **Q** are respectively **8** and **6** units. The cost (in ₹) of transportation per unit is given below.

Cost (in ₹)			
To⟍From	**A**	**B**	**C**
P	160	100	150
Q	100	120	100

Based on the above information, answer the following questions:

(i) Formulate the objective function and the constraints of the above Linear programming problem. [**2**]

(ii) How many units should be transported from each factory to each depot in order that the transportation cost is minimum? [**2**]

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