Strictly Confidential — (For Internal and Restricted Use Only)

Secondary School Certificate Examination

July'2018

Marking Scheme — Mathematics 30/3 (Compt.)

General Instructions:

1. The Marking Scheme provides general guidelines to reduce subjectivity in the marking. The answers given in the Marking Scheme are suggested answers. The content is thus indicative. If a student has given any other answer which is different from the one given in the Marking Scheme, but conveys the meaning, such answers should be given full weightage.

2. Evaluation is to be done as per instructions provided in the marking scheme. It should not be done according to one's own interpretation or any other consideration — Marking Scheme should be strictly adhered to and religiously followed.

3. Alternative methods are accepted. Proportional marks are to be awarded.

4. In question (s) on differential equations, constant of integration has to be written.

5. If a candidate has attempted an extra question, marks obtained in the question attempted first should be retained and the other answer should be scored out.

6. A full scale of marks - 0 to 100 has to be used. Please do not hesitate to award full marks if the answer deserves it.

7. Separate Marking Scheme for all the three sets has been given.

8. As per orders of the Hon'ble Supreme Court. The candidates would now be permitted to obtain photocopy of the Answer book on request on payment of the prescribed fee. All examiners/ Head Examiners are once again reminded that they must ensure that evaluation is carried out strictly as per value points for each answer as given in the Marking Scheme.

QUESTION PAPER CODE 30/3

EXPECTED ANSWER/VALUE POINTS

SECTION A

1. $\frac{a^3}{A^3} = \frac{1}{27}$ $\frac{1}{2}$

$\Rightarrow \quad \frac{a}{A} = \frac{1}{3}$

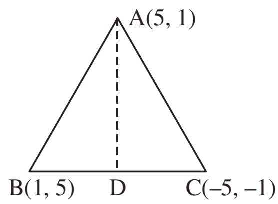
Ratio of sufrace area = $\frac{6a^2}{6A^2} = \left(\frac{1}{3}\right)^2 = \frac{1}{9}$ $\frac{1}{2}$

2. Let α and $\frac{1}{\alpha}$ be the root

$\therefore \quad \alpha \cdot \frac{1}{\alpha} = \frac{k}{5} = 1$ $\frac{1}{2}$

$\Rightarrow \quad k = 5$ $\frac{1}{2}$

3. Coordinates of D are (–1, 2) $\frac{1}{2}$



$AD = \sqrt{(5+1)^2 + (1+2)^2}$

$= \sqrt{37}$ units $\frac{1}{2}$

4. Solving for x and y and getting x = 3, y = 1 $\frac{1}{2}$

$\therefore \quad a = 3,\ b = 1$ $\frac{1}{2}$

5. $\frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta QRP)} = \left(\frac{BC}{RP}\right)^2$ $\frac{1}{2}$

$\Rightarrow \frac{9}{4} = \left(\frac{15}{PR}\right)^2 \Rightarrow PR = 10 \text{ cm}$ $\frac{1}{2}$

6. For writing $\frac{6\sqrt{5}+6\sqrt{5}}{2\sqrt{5}}$ $\frac{1}{2}$

$= 6$ which is rational $\frac{1}{2}$

SECTION B

7. Let r be the radii of bases of cylinder and cone and h be the height

Slant height of cone $= \sqrt{r^2 + h^2}$ $\frac{1}{2}$

$\therefore \frac{2\pi rh}{\pi r\sqrt{r^2+h^2}} = \frac{8}{5}$ $\frac{1}{2}$

$\frac{h}{\sqrt{r^2+h^2}} = \frac{4}{5}$

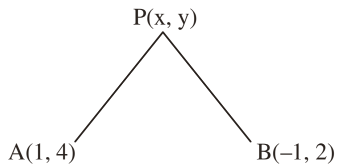
$\Rightarrow \frac{h^2}{r^2+h^2} = \frac{16}{25}$

$\Rightarrow 25h^2 = 16r^2 + 16h^2$

$\Rightarrow 9h^2 = 16r^2$ $\frac{1}{2}$

$\Rightarrow \frac{r^2}{h^2} = \frac{9}{16} \Rightarrow \frac{r}{h} = \frac{3}{4}$ $\frac{1}{2}$

8. $PA = PB \Rightarrow PA^2 = PB^2$

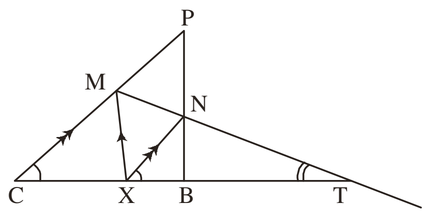


$\Rightarrow \quad (x - 1)^2 + (y - 4)^2 = (x + 1)^2 + (y - 2)^2$ 1

$\Rightarrow \quad x^2 + 1 - 2x + y + 16 - 8y = x^2 + 1 + 2x + y^2 + 4 - 4y$ $\frac{1}{2}$

$\Rightarrow \quad x + y - 3 = 0$ $\frac{1}{2}$

9. $\Delta TXN \sim \Delta TCM$ $\frac{1}{2}$



$\Rightarrow \quad \frac{TX}{TC} = \frac{XN}{CM} = \frac{TN}{TM}$

$\Rightarrow \quad TX \times TM = TC \times TN \qquad ....(i)$

Again, $\Delta TBN \sim \Delta TXM$ $\frac{1}{2}$

$\Rightarrow \quad \frac{TB}{TX} = \frac{BN}{XM} = \frac{TN}{TM}$

$\Rightarrow \quad TM = \frac{TN \times TX}{TB} \qquad ...(ii)$ $\frac{1}{2}$

using (*ii*) in (*i*), we get

$$TX^2 \times \frac{TN}{TB} = TC \times TN$$

$\Rightarrow \quad TX^2 = TC \times TB$ $\frac{1}{2}$

10. Let $2+\sqrt{3}$ be a rational number.

$\Rightarrow \quad 2+\sqrt{3} = \frac{p}{q}, \quad p, q \in I, q \neq 0$ $\frac{1}{2}$

$\Rightarrow \quad \sqrt{3} = \frac{p}{q} - 2 = \frac{p-2q}{q}$ $\frac{1}{2}$

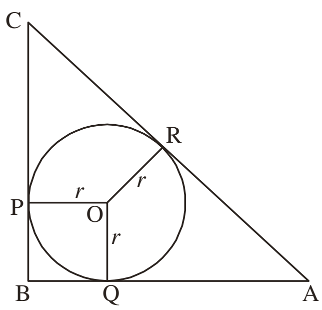
$\frac{p-2q}{q}$ is rational $\Rightarrow \sqrt{3}$ is rational number $\frac{1}{2}$

which is a contraduction

$2+\sqrt{3}$ is irrational number $\frac{1}{2}$

11. $AC = \sqrt{AB^2 + BC^2}$

$= \sqrt{14^2 + 48^2} = \sqrt{2500} = 50 \text{ cm}$ $\frac{1}{2}$



$\angle OQB = 90° \Rightarrow$ OPBQ is a square

$\Rightarrow \quad BQ = r, QA = 14 - r = AR$ $\frac{1}{2}$

Again PB = r,

$PC = 48 - r \Rightarrow RC = 48 - r$ $\frac{1}{2}$

$AR + RC = AC \Rightarrow 14 - r + 48 - r = 50$

$\Rightarrow \quad r = 6 \text{ cm}$ $\frac{1}{2}$

12. $A + B + C = 180°$

$$\Rightarrow \quad \frac{A+B}{2} = 90° - \frac{C}{2}$$ 1

$$\Rightarrow \quad \text{cosec}\left(\frac{A+B}{2}\right) = \text{cosec}\left(90° - \frac{C}{2}\right) = \sec\frac{C}{2}$$ 1

SECTION C

13. Construction of ΔABC with sides 6 cm, 8 cm, 4 cm. 1

Construction of similar triangle 2

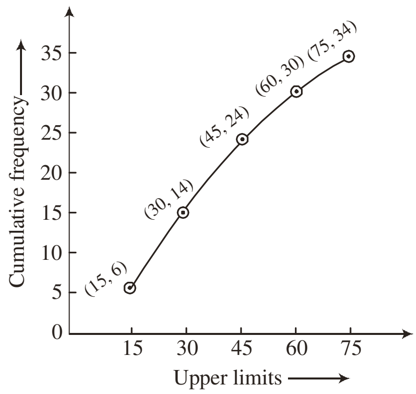
14. $\sin(A + 2B) = \frac{\sqrt{3}}{2} \Rightarrow A + 2B = 60°$ 1

$\cos(A + 4B) = \Rightarrow A + 4B = 90°$ 1

Solving, we get $A = 30°$, $B = 15°$ $\frac{1}{2}+\frac{1}{2}$

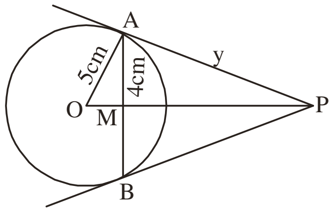
15.

Classes	Frequency	Classes	Cumulative frequency	
0-15	6	Less than 15	6	
15-30	8	Less than 30	14	
30-45	10	Less than 45	24	1
45-60	6	Less than 60	30	
60-75	4	Less than 75	34	



2

16. AB = 8 cm $\Rightarrow$ AM = 4 cm



$\therefore$ OM = $\sqrt{5^2 - 4^2} = 3\,\text{cm}$

Let AP = y cm, PM = x cm

$\therefore$ ΔOPP is a right angle triangle

$\therefore$ $OP^2 = OA^2 = AP^2$

$(x + 3)^2 = y^2 + 25$

$\Rightarrow$ $x^2 + 9 + 6x = y^2 + 25$ ...(*i*) 1

Also $x^2 + 4^2 = y^2$ ...(*ii*) 1

$\Rightarrow$ $x^2 + 6x + 9 = x^2 + 16 + 25$

$\Rightarrow$ $6x = 32 \Rightarrow x = \frac{32}{6}$ i.e. $\frac{16}{3}$cm

$\therefore$ $y^2 = x^2 + 16 = \frac{256}{9} + 16 = \frac{400}{9}$

$\Rightarrow$ $y = \frac{20}{3}$ cm or $6\frac{2}{3}$cm 1

OR

Correct given, to prove, figure and construction $\frac{1}{2} \times 4 = 2$

Correct proof 1

17. 867 = 255 × 3 + 102 1

255 = 102 × 2 + 51 1

$102 = 51 \times 2 + 0$ $\frac{1}{2}$

$\Rightarrow$ HCF = 51 $\frac{1}{2}$

18. Distance travelled by short hand in 48 hours = $4 \times 2\pi \times 4$ cm = 32π cm 1

Distance travelled by long hand in 48 hours = $48 \times 2\pi \times 6$cm = 576π cm 1

Total distance travelled = $(32\pi + 576\pi)$cm

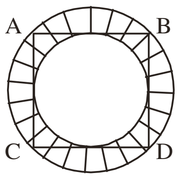
= 608π cm 1

OR

Radius of inner circle = 5 cm $\frac{1}{2}$

Radius of outer circle = $5\sqrt{2}$ cm 1

Required area = Area of outer circle – Area of inner circle 1



$\Rightarrow$ $[(5\sqrt{2})^2 - 5^2] = 25\pi\ cm^2$ $\frac{1}{2}$

19. Here, $S_n = 3n^2 + 5n$

$\Rightarrow$ $S_1 = 3.1^2 + 5.1 = 8 = a_1$ $\frac{1}{2}$

$S_2 = 3.2^2 + 5.2 = 22 = a_1 + a_2$

$a_2 = 22 - 8 = 14 \Rightarrow d = 6$ 1

$t_k = 164 \Rightarrow 8 + (k-1)6 = 164$ $\frac{1}{2}$

$\Rightarrow$ k = 27 1

20. Let two parts be x and 27 – x 1

$\therefore \quad \frac{1}{x} + \frac{1}{27-x} = \frac{3}{20}$ 1

$\Rightarrow \quad x^2 - 27x + 150 = 0$ 1

$\Rightarrow \quad (x-15)(x-12) = 0$

$\Rightarrow \quad x = 12 \text{ or } 15$ 1

$\therefore$ The two parts are 12 and 15

21. $\frac{1+\tan^2 A}{1+\cot^2 A} = \frac{1+\tan^2 A}{1+\frac{1}{\tan^2 A}} = \tan^2 A$ $1\frac{1}{2}$

$\left(\frac{1-\tan A}{1-\cot A}\right)^2 = \left(\frac{1-\tan A}{\frac{\tan A - 1}{\tan A}}\right)^2 = (-\tan A)^2 = \tan^2 A$ $1\frac{1}{2}$

Hence $\frac{1+\tan^2 A}{1+\cot^2 A} = \left(\frac{1-\tan A}{1-\cot A}\right)^2 = \tan^2 A$

OR

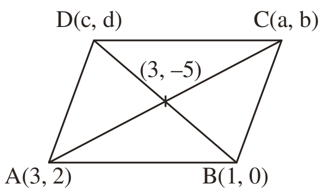
$\frac{\cos 58°}{\sin 32°} + \frac{\sin 22°}{\cos 68°} - \frac{\cos 38° \operatorname{cosec} 52°}{\sqrt{3}(\tan 18° \tan 35° \tan 60° \tan 72° \tan 55°)}$

$= \left(\frac{\cos 58°}{\sin (90-58°)} + \frac{\sin 22°}{\cos (90-22°)}\right) - \frac{\cos 38° \operatorname{cosec} (90-38)°}{\sqrt{3}(\tan 18° \tan 35° \cdot \sqrt{3} \cdot \cot 18° \cot 35°)}$ 1+1

$= 1 + 1 - \frac{\cos 38° \sec 38°}{3.1}$

$= 2 - \frac{1}{3} = \frac{5}{3}$ 1

22. Let the coordinates of C and D be (a, b) and (c, d)



$\therefore \quad \frac{3+a}{2} = 2 \Rightarrow a = 1$ 1

and $\frac{2+b}{2} = -5 \Rightarrow b = -12$

Also $\frac{c+1}{2} = -5 \Rightarrow c = 3$ 1

and $\frac{d+0}{2} = -5 \Rightarrow d = -10$

$\therefore$ Coordinate of C and D are (1, –12) and (3, –10) 1

OR

Ar (ΔABC) = 4

$\Rightarrow \quad \frac{1}{2}[x(4-5)+4(5-3)+3(3-4)] = 4$ $1\frac{1}{2}$

$\Rightarrow \quad (-x+5) = 8$

$\Rightarrow \quad -x+5 = 8$ 1

$\Rightarrow \quad x = -3$ $\frac{1}{2}$

SECTION D

23. Let the speed of faster train be x km/hr

$\therefore$ Speed of slower train = (x – 10) km/hr $\frac{1}{2}$

$\frac{200}{x-10} - \frac{200}{x} = 1$ $1\frac{1}{2}$

$\Rightarrow \quad x^2 - 10x - 2000 = 0$ 1

$\Rightarrow \quad (x-50)(x+40) = 0$ 1

$x = 50, -40$ rejected

$\therefore$ Speed of faster train = 50 km / hr
Speed of slower train = 40 km / hr $\Big\}$ 1

OR

$$\frac{1}{a+b+x} = \frac{1}{a}+\frac{1}{b}+\frac{1}{x}$$

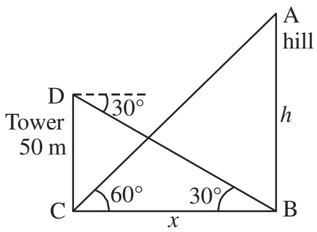
$$\Rightarrow \quad \frac{1}{a+b+c}-\frac{1}{x} = \frac{1}{a}+\frac{1}{b}$$

$$\frac{-(a+b)}{x(a+b+x)} = \frac{a+b}{ab}$$ 2

$\Rightarrow \quad x^2 + (a + b)x + ab = 0$ 1

$(x + a)(x + b) = 0 \Rightarrow x = -a, -b$ 1

24.



Correct figure 1

In ΔABC, $\frac{h}{x} = \tan 60°$

$\Rightarrow \quad h = x\sqrt{3}$ 1

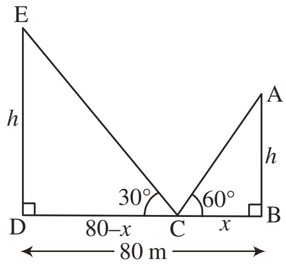
In ΔBCD, $\frac{50}{x} = \tan 30°$

$\Rightarrow \quad x = 50\sqrt{3}$ 1

$\therefore \quad h = 150$

$\therefore$ height of hill = 150 m 1

OR



Correct figure 1

In ΔABC, $\frac{h}{x} = \tan 60°$

$\Rightarrow \quad h = x\sqrt{3}$ ....(1) 1

In ΔECD, $\frac{h}{80 - x} = \tan 30°$

$\Rightarrow \quad h\sqrt{3} = 80 - x$ 1

From (1), $x\sqrt{3} \times \sqrt{3} = 80 - x$

$\Rightarrow \quad x = 20$

$\therefore \quad h = 20\sqrt{3}$

$\therefore$ height of poles $= 20\sqrt{3}$m 1

Distances of poles from the point are 20 m and 60 m

25. $p(x) = 3x^4 - 15x^3 + 13x + 25x - 30$

$x - \sqrt{\frac{5}{3}}$ and $x + \sqrt{\frac{5}{3}}$ are factors of p(x)

$\Rightarrow \quad x^2 - \frac{5}{3}$ or $\frac{(3x^2 - 5)}{3}$ is a factor of p(x) 1

$p(x) = \frac{(3x^2 - 5)}{3}(x^2 - 5x + 6)$ 2

$= \frac{1}{3}(3x^2 - 5)(x - 3)(x - 2)$

$\therefore$ Zeroes of p(x) are $\sqrt{\frac{5}{3}}, -\sqrt{\frac{5}{3}}, 2$ and 3 — 1

26. Surface area of bucket $= \pi(r_1 + r_2)l + \pi r_1^2$

$l = \sqrt{h^2 + (r_2 - r_1)^2} = \sqrt{20^2 + (36-21)^2}$

$= \sqrt{625} = 25$ cm — $\frac{1}{2}$

$\therefore$ Surface area of 1 bucket $= \frac{22}{7}[(36+21)\times 25 + 21^2]$

$= \frac{22}{7} \times 1866$ cm^2 — 1

Surface area of 10 buckets $= \frac{22}{7} \times 18660$ cm^2 — $\frac{1}{2}$

Cost of aluminium sheet = ₹ $\frac{22}{7} \times \frac{18660 \times 42}{100}$ — 1

= ₹ 24631.20

Any relevant comment — 1

27.

Classes	Frequency	x_i	$f_i x_i$
10-20	4	15	60
20-30	8	25	200
30-40	10	35	350
40-50	12	45	540
50-60	10	55	550
60-70	4	65	260
70-80	2	75	150
Total	50		2110

2

$$\bar{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{2110}{50} = 42.2$$ 1

40-50 is modal class

$$\text{Mode} = l + \frac{(f_1 - f_0)}{2f_1 - f_0 - f_2} \times h$$

$$= 40 + \frac{12-10}{24-10-10} \times 10 = 45$$ 1

28. For infinitely many solutions.

$$\frac{3}{m+n} = \frac{4}{2(m-n)} = \frac{-12}{-(5m-1)}$$ 1

$$\frac{3}{m+n} = \frac{4}{2(m-n)} \Rightarrow m - 5n = 0 \quad ...(1)$$ 1

$$\frac{4}{2(m-n)} = \frac{12}{5m-1} \Rightarrow m - 6n = -1 \quad ...(2)$$ 1

Solving (1) and (2) we get, $m = 5, n = 1$ 1

29. (*i*) Prime numbers from 1 to 20 are 2, 3, 5, 7, 11, 13, 17, 19 i.e. 8

P(prime number) = $\frac{8}{20}$ or $\frac{2}{5}$ $1\frac{1}{2}$

(*ii*) Composite number from 1 to 20 are

4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20 i.e. 11

P(Composite number) = $\frac{11}{20}$ $1\frac{1}{2}$

(*iii*) Number divisible by 3 from 1 to 20 are

3, 6, 9, 12, 15, 18 i.e 6

P(number divisible by 3) = $\frac{6}{20}$ or $\frac{3}{10}$ 1

OR

Total number of cards = 52 – 3 = 49

(*i*) P(spade) = $\frac{13}{49}$ 1

(*ii*) P(black king) = $\frac{1}{49}$ 1

(*iii*) P(club) = $\frac{10}{49}$ 1

(*iv*) P(Jack) = $\frac{3}{49}$ 1

30. Correct figure, given to prove and construction $\frac{1}{2} \times 4 = 2$

Correct proof 2