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F/30(B)

Secondary School Examination

March 2018

Marking Scheme — Mathematics 30(B)

General Instructions:

1. The Marking Scheme provides general guidelines to reduce subjectivity in the marking. The answers given in the Marking Scheme are suggested answers. The content is thus indicative. If a student has given any other answer which is different from the one given in the Marking Scheme, but conveys the meaning, such answers should be given full weightage

2. Evaluation is to be done as per instructions provided in the marking scheme. It should not be done according to one's own interpretation or any other consideration — Marking Scheme should be strictly adhered to and religiously followed.

3. Alternative methods are accepted. Proportional marks are to be awarded.

4. If a candidate has attempted an extra question, marks obtained in the question attempted first should be retained and the other answer should be scored out.

5. A full scale of marks - 0 to 80 has to be used. Please do not hesitate to award full marks if the answer deserves it.

6. As per orders of the Hon'ble Supreme Court. The candidates would now be permitted to obtain photocopy of the Answer book on request on payment of the prescribed fee. All examiners/Head Examiners are once again reminded that they must ensure that evaluation is carried out strictly as per value points for each answer as given in the Marking Scheme.

QUESTION PAPER CODE 30(B)

EXPECTED ANSWER/VALUE POINTS

SECTION A

1. Roots are equal

$\Rightarrow \quad 4(k-5)^2 - 8(k-5) = 0$ ½

$\Rightarrow \quad 4(k-5)(k-7) = 0$

$\Rightarrow \quad k \neq 5, k = 7$ ½

2. Given $\sqrt{(2-10)^2 + (-3-y)^2} = 10$

$\Rightarrow \quad 64 + 9 + y^2 + 6y = 100$ ½

$\Rightarrow \quad y^2 + 6y - 27 = 0$

$\Rightarrow \quad (y+9)(y-3) = 0$

$\Rightarrow \quad y = -9$ or $y = 3.$ ½

3. $3125 = 5 \times 5 \times 5 \times 5 \times 5 = 5^5 \times 2^0$ ½

$\therefore \quad \frac{13}{3125}$ is terminating decimal expansion. ½

4. $a = \frac{1}{k}, \quad d = \frac{1+k}{k} - \frac{1}{k} = 1$ ½

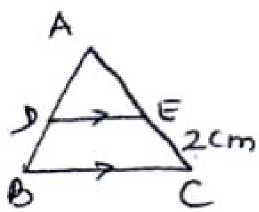
$\therefore \quad a_m = \frac{1}{k} + (m-1) \times 1 = \frac{1}{k} + m - 1$ or $\frac{1+(m-1)k}{k}$ ½

5. $\sin\theta + \cos\theta = \sqrt{2}\sin\theta$

$\Rightarrow \quad \frac{\cos\theta}{\sin\theta} = \sqrt{2} - 1$ ½

$\Rightarrow \quad \cot\theta = \sqrt{2} - 1$ ½

6.



$\frac{AB}{BD} = 4 \Rightarrow \frac{AB - BD}{BD} = 3$

$\Rightarrow \quad \frac{AD}{DB} = 3 = \frac{AE}{EC}$ ½

$\Rightarrow \quad AE = 6$ cm. ½

SECTION B

7. Let the number of blue balls be n.

$\therefore$ Total number of balls = 5 + n — ½

$\because$ Prob (drawing a blue ball) = 3 × Prob (drawing a red ball)

$\Rightarrow \quad \frac{n}{5+n} = 3 \times \frac{5}{5+n}$ — 1

$\Rightarrow$ n = 15. — ½

8. $a + 4d = 13 \quad ...(1)$

$a + 14d = -17 \quad ...(2)$ — ½

Solving to get $d = -3$ and $a = 25$ — ½

$\therefore \quad S_{21} = \frac{21}{2}[50 + 20 \times (-3)]$

$= -105$ — 1

9. $867 = 3 \times 255 + 102$

$255 = 2 \times 102 + 51$

$102 = 2 \times 51 + 0$ — 1½

$\therefore$ HCF of 867 and 255 = 51 — ½

10. $(3)^2 + (k-2)^2 = k^2 + (5-2)^2$ — 1

$\Rightarrow \quad 9 + k^2 - 4k + 4 = k^2 + 9$

$\Rightarrow \quad k = 1$ — 1

11. For infinitely many solutions

$\frac{2}{4} = \frac{3}{a} = \frac{7}{14}$ — 1

$\Rightarrow \quad a = 6.$ — 1

12. Total number of cards = 52.

(i) Prob (getting a red king) = $\frac{2}{52}$ or $\frac{1}{26}$ — 1

(ii) Prob (getting a queen or a jack) = $\frac{8}{52}$ or $\frac{2}{13}$. — 1

SECTION C

13. Let a be any positive odd integer and b = 4

Using Euclid's division lemma

$a = 4q + r,\ q \geq 0,\ 0 \leq r < 4$ i.e., $r = 0, 1, 2, 3$ 1

$\Rightarrow\ a = 4q,\ 4q + 1,\ 4q + 2$ or $4q + 3$. 1

Since a is odd

therefore $a = 4q + 1$ or $a = 4q + 3$ 1

14. Let the unit's digit be x and ten's digit be y.

$\Rightarrow\ y = 2x$ ...(1) 1

Also $10x + y = (10y + x) - 36$ 1

$\Rightarrow\ 9x - 9y = -36$

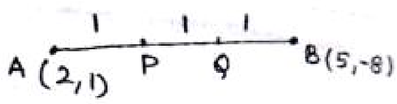
$\Rightarrow\ x - y = -4$ ...(2)

Solving (1) and (2) to get $x = 4$ and $y = 8$. ½

∴ The original number was 84. ½

15. AP : PB = 1 : 2

∴ Point P is $\left(\frac{5+4}{3}, \frac{-8+2}{3}\right)$ ½



i.e., point P is (3, –2) 1

Since P lies on the line $2x - y + k = 0$

$\Rightarrow\ 6 + 2 + k = 0$ 1

$\Rightarrow\ k = -8$ ½

OR

Let the point P be (2y, y) 1

Now PQ = PR

$\Rightarrow\ (2y - 2)^2 + (y + 5)^2 = (2y + 3)^2 + (y - 6)^2$ 1

$\Rightarrow\ 5y^2 + 29 + 2y = 5y^2 + 45$

$\Rightarrow\ y = 8$ ½

∴ Coordinates of point P are (16, 8). ½

16. Let $p(x) = 2x^3 + x^2 - 5x + 2$

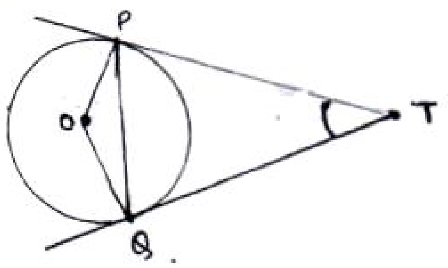
$p(1) = 2(1)^3 + (1)^2 - 5(1) + 2 = 2 + 1 - 5 + 2 = 0$ 1

$p\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^2 - 5\times\frac{1}{2} + 2 = \frac{1}{4} + \frac{1}{4} - \frac{5}{2} + 2 = 0$ 1

$p(-2) = 2(-2)^3 + (-2)^2 - 5(-2) + 2 = -16 + 4 + 10 + 2 = 0$ 1

$\therefore$ $1, \frac{1}{2}, -2$ are zeroes of polynomial p(x).

17.



$\left.\begin{array}{l} OP \perp PT \Rightarrow \angle OPT = 90° \\ OQ \perp QT \Rightarrow \angle OQT = 90° \end{array}\right\}$ 1

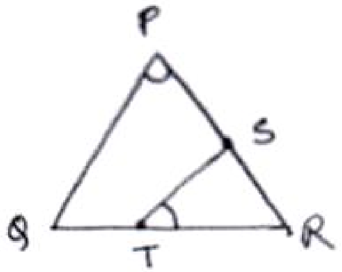
Since OQTP is a quadrilateral

$\therefore$ $\angle O + \angle P + \angle T + \angle Q = 360°$ 1

$\Rightarrow$ $\angle O + \angle T = 360° - 180° = 180°$

Hence, $\angle O$ and $\angle T$ are supplementary angles. 1

18.



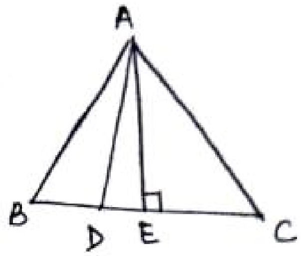
In Δ^S PQR and STR

$\angle P = \angle RTS$ (given) 1

$\angle R = \angle R$ (common angle) ½

$\therefore$ $\Delta RPQ \sim \Delta RTS$ (AA similarity) 1½

OR



Let AB = BC = CA = x

$\Delta AEB \cong \Delta AEC$ (RHS congruence rule)

$\Rightarrow$ $BE = EC = \frac{x}{2}$

Also DE = BE – BD

$= \frac{x}{2} - \frac{x}{3}$

$\Rightarrow$ $DE = \frac{x}{6}$ ½

Now $AB^2 = AE^2 + BE^2$

and $AD^2 = AE^2 + DE^2$

$\Rightarrow \; AB^2 - AD^2 = BE^2 - DE^2$ 1

$\Rightarrow \; AD^2 = x^2 - \frac{x^2}{4} + \frac{x^2}{36}$ 1

$= \frac{28}{36}x^2$ ½

$\Rightarrow \; 9AD^2 = 7AB^2$

19. Simplifying LHS and RHS simultaneously

$\frac{\sin\theta}{1+\cos\theta} - \frac{1}{\sin\theta} = \frac{1}{\sin\theta} - \frac{\sin\theta}{1-\cos\theta}$ 1

$\Rightarrow \; \sin\theta\left(\frac{1}{1+\cos\theta} + \frac{1}{1-\cos\theta}\right) = \frac{2}{\sin\theta}$ 1

$\Rightarrow \; \sin\theta \times \frac{2}{\sin^2\theta} = \frac{2}{\sin\theta}$ which is true. 1

OR

$m^2 - n^2 = (\tan\theta + \sin\theta)^2 - (\tan\theta - \sin\theta)^2$

$= \tan^2\theta + \sin^2\theta + 2\tan\theta\sin\theta - \tan^2\theta - \sin^2\theta + 2\tan\theta\sin\theta$

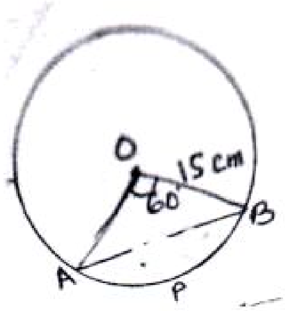
$= 4\tan\theta\sin\theta$ 1

$4\sqrt{mn} = 4\sqrt{(\tan^2\theta - \sin^2\theta)} = 4\sqrt{\frac{\sin^2\theta}{\cos^2\theta}(1-\cos^2\theta)}$ 1

$= 4\tan\theta\sin\theta.$ ½

Therefore $m^2 - n^2 = 4\sqrt{mn}$ ½

20. Area of minor segment = Area sector APBO − Area ΔOAB



Area of sector OAPB $= \frac{60°}{360°} \times (3.14)r^2$

$= \frac{1}{6} \times 3.14 \times 15 \times 15 \text{ cm}^2$ 1

Area of ΔOAB $= \frac{\sqrt{3}}{4} \times 15 \times 15 \text{ cm}^2$ ($\because$ OAB is an equilateral triangle.) ½

Hence area of minor segment $= 15 \times 15\left(\frac{3.14}{6} - \frac{1.73}{4}\right)$

$= 20.44 \text{ cm}^2$ ½

Area of major segment $= (3.14 \times 15 \times 15 - 20.44) \text{ cm}^2$

$= 686.06 \text{ cm}^2$ 1

21. Volume of water raised in vessel $= \pi \times r^2 \times \frac{32}{9} \text{ cm}^3$ 1

$\Rightarrow$ Volume of sphere $= \frac{4}{3}\pi \times r_1^3 = \pi r^2 \times \frac{32}{9}$ $(r_1 = 6 \text{ cm})$ 1

$\Rightarrow$ $r^2 = 81$

$\Rightarrow$ $r = 9$ cm and diameter 18 cm. ½+½

OR

Here $h = \frac{2}{3} \times 2r = \frac{4}{3}r$ 1

Given Volume of cylinder = Volume of sphere

$\Rightarrow$ $\pi \times r^2 \times \frac{4}{3}r = \frac{4}{3}\pi \times 4 \times 4 \times 4$ 1

$\Rightarrow$ $r = 4$ cm. 1

22.

Daily Income	f	x	$u = \frac{x-150}{20}$	fu
100–120	12	110	–2	–24
120–140	14	130	–1	–14
140–160	8	150	0	0
160–180	6	170	1	6
180–200	10	190	2	20
	50			–12

Correct Table 1

Mean $\bar{x} = a + h\frac{\Sigma fu}{\Sigma f} = 150 + \frac{20}{50} \times (-12)$

$= \text{Rs. } 145.20$ 1

Modal class = 120 – 140

Mode $= l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$

$= 120 + \frac{14-12}{28-12-8} \times 20$

$= \text{Rs. } 125$ 1

SECTION D

23. Let the tap of smaller diameter fills the tank in x hrs.
∴ Tap of larger diameter fills the tank in (x – 9) hrs. 1

According to the statements

$$\frac{1}{x} + \frac{1}{x-9} = \frac{1}{6}$$ 1

$\Rightarrow$ $6(2x - 9) = x(x - 9)$

$\Rightarrow$ $x^2 - 21x + 54 = 0$ 1

$\Rightarrow$ $(x - 18)(x - 3) = 0$

$x \neq 3 \quad \therefore x = 18$ ½

taps can fill the tank seperately in 18 hrs and 9 hrs respectively. ½

OR

$$\frac{(x+1)^2-(x-1)^2}{x^2-1}=\frac{5}{6}$$ 1

$\Rightarrow \quad 6(2x + 2x) = 5(x^2 - 1)$ 1

$\Rightarrow \quad 5x^2 - 24x - 5 = 0$

$\Rightarrow \quad (5x + 1)(x - 5) = 0$ 1

$\Rightarrow \quad x = \frac{-1}{5},\ x = 5.$ 1

24. Correct Given, To prove, Construction $\frac{1}{2}\times 3 = 1\frac{1}{2}$

Correct Proof $2\frac{1}{2}$

OR

Correct Given, To prove, Construction $\frac{1}{2}\times 3 = 1\frac{1}{2}$

Correct proof $2\frac{1}{2}$

25. Steps of Construction of ΔABC 2

Steps of Construction of triangle similar to ΔABC. 2

26. $S_n = 5n^2 + 3n$

$S_1 = a = 5 + 3 = 8 \Rightarrow a = 8$ 1

$S_2 = 2a + d = 20 + 6 = 26 \Rightarrow 2a + d = 26$

$\Rightarrow \quad d = 10.$ 1

$a_m = a + (m - 1)d = 168$ or $8 + (m - 1) \times 10 = 168$

$\Rightarrow \quad m = 17$ 1

Therefore $a_{20} = 8 + 190 = 198$ 1

OR

$$\left.\begin{aligned} a + 3d &= 11 \quad ...(1) \\ a + 29d &= 89 \quad ...(2) \end{aligned}\right\}$$ 1

Solving (1) and (2) to get $a = 2, d = 3$ $\frac{1}{2}+\frac{1}{2}$

$\therefore$ A.P. is 2, 5, 8, ... 1

$a_{23} = 2 + 22 \times 3 = 68$ 1

27. LHS $= \dfrac{\sin^2 A-(1-\cos A)^2}{\sin A\,(1-\cos A)} \cdot \dfrac{\cos^2 A-(1-\sin A)^2}{\cos A\,(1-\sin A)}$ 1

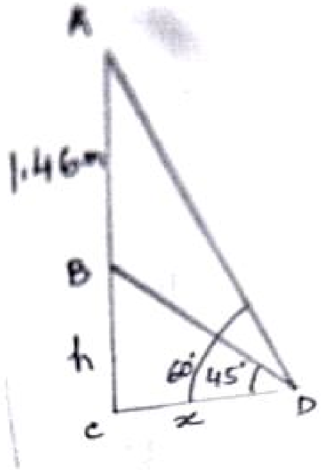
$= \dfrac{\sin^2 A-1-\cos^2 A+2\cos A}{\sin A(1-\cos A)} \cdot \dfrac{\cos^2 A-1-\sin^2 A+2\sin A}{\cos A(1-\sin A)}$ 1

$= \dfrac{-2\cos^2 A+2\cos A}{\sin A(1-\cos A)} \cdot \dfrac{-2\sin^2 A+2\sin A}{\cos A(1-\sin A)}$ 1

$= \dfrac{2\cos A}{\sin A} \cdot \dfrac{2\sin A}{\cos A}$ ½

= 4 = RHS. ½

28.



Let AB represents statue and BC represents pedestal

$\tan 45° = 1 = \dfrac{h}{x}$

$\Rightarrow$ h = x 1

$\tan 60° = \sqrt{3} = \dfrac{h+1.46}{x}$ 1½

$\Rightarrow$ $h\sqrt{3} - h = 1.46$

$\Rightarrow$ $h = \dfrac{1.46}{0.73} = 2$ m. 1½

29. Volume of each drum $= \dfrac{22}{7} \times 0.7 \times 0.7 \times 2$

$= 3.08\ m^3$ 1

Cost of each drum = Rs. 350 × 3.08

= Rs. 1078 1

Therefore cost of 3 drums = Rs. 3234 1

Any relevant value. 1

30.

Class	frequency	Cumulative frequency
0–10	2	2
10–20	5	7
20–30	x	7 + x
30–40	12	19 + x
40–50	17	36 + x
50–60	20	56 + x
60–70	y	56 + x + y
70–80	9	65 + x + y
80–90	7	72 + x + y
90–100	4	76 + x + y

For Correct Table 1

Total frequency = 100

$\Rightarrow$ $76 + x + y = 100$

$\Rightarrow$ $x + y = 24$ ...(1) ½

Median = 52.5

$\therefore$ Median class is 50–60 ½

$\Rightarrow$ $50 + \frac{10}{20}(50 - 36 - x) = 52.5$ 1

$\Rightarrow$ $x = 9$ ½

and $y = 15$ ½