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II

Total No. of Questions – 24 **Regd. No.**

Total No. of Printed Pages - 4

Part – III
MATHEMATICS, Paper – II(A)
(English Version)

Time : 3 Hours] **[Max. Marks : 75**

Note : This question paper consists of **three** Sections – **A**, **B** and **C**.

SECTION – A
10 × 2 = 20

I. Very Short Answer Type questions.

(i) Attempt **all** the questions.

(ii) Each question carries **two** marks.

1. If $(\sqrt{3} + i)^{100} = 2^{99} (a + ib)$, then show that $a^2 + b^2 = 4$.

2. If $z = 2 - 3i$, then show that $z^2 - 4z + 13 = 0$.

3. If 1, w, w^2 are the cube roots of unity, then find the value of $(1 - w + w^2)^5 + (1 + w - w^2)^5$.

4. Find the values of m, for which the equation $x^2 - 15 - m(2x - 8) = 0$ have equal roots.

5. If the product of the roots of $4x^3 + 16x^2 - 9x - a = 0$ is 9, then find a.

6. Find the number of ways of arranging the letters of the word INTERMEDIATE.

7. If ${}^nC_5 = {}^nC_6$, then find ${}^{13}C_n$.

8. Find the 6^{th} term in the expansion of $\left(\frac{2x}{3}+\frac{3y}{2}\right)^9$.

9. Find the mean deviation about the median for the following data :

   4, 6, 9, 3, 10, 13, 2.

10. The mean and variance of a binomial distribution are 4 and 3 respectively, fix the distribution and find $P(X \geq 1)$.

SECTION – B

$5 \times 4 = 20$

II. Short Answer Type questions.

(i) Attempt any **five** questions.

(ii) Each question carries **four** marks.

11. If $x + iy = \frac{1}{1+\cos\theta+i\sin\theta}$, then show that $4x^2 - 1 = 0$

12. If x is real, prove that $\frac{x}{x^2-5x+9}$ lies between $-\frac{1}{11}$ and 1.

13. If the letters of the word MASTER are permuted in all possible ways and the words thus formed are arranged in the dictionary order, then find the rank of the word MASTER.

14. Simplify : ${}^{34}C_5 + \sum_{r=0}^{4} {}^{(38-r)}C_4$.

15. Resolve : $\frac{x^3}{(x-1)(x+2)}$ into partial fractions.

16. A, B, C are three horses in a race. The probability of A to win the race is twice that of B, and the probability of B is twice that of C. What are the probabilities of A, B and C to win the race ?

17. A speaks truth in 75% of the cases and B in 80% cases. What is the probability that their statements about an incident do not match.

SECTION – C $5 \times 7 = 35$

III. Long Answer Type questions :

(i) Attempt any **five** questions.

(ii) Each question carries **seven** marks.

18. If α, β are the roots of the equation $x^2 - 2x + 4 = 0$, then for any $n \in N$, show that $\alpha^n + \beta^n = 2^{n+1} \cdot \cos\left(\frac{n\pi}{3}\right)$

19. Solve $18x^3 + 81x^2 + 121x + 60 = 0$, given that one root is equal to half the sum of the remaining roots.

20. If the co-efficient of x^{10} in the expansion of $\left(ax^2 + \frac{1}{bx}\right)^{11}$ is equal to the co-efficient of x^{-10} in the expansion of $\left(ax - \frac{1}{bx^2}\right)^{11}$, then find the relation between a and b where a and b are real.

21. If $x = \frac{1.3}{3.6} + \frac{1.3.5}{3.6.9} + \frac{1.3.5.7}{3.6.9.12} + \ldots\ldots \infty$, then prove that $9x^2 + 24x = 11$.

22. Calculate the variance and standard deviation for the following discrete frequency distribution :

x_i	4	8	11	17	20	24	32
f_i	3	5	9	5	4	3	1

23. State and prove Baye's Theorem.

24. The probability distribution of a random variable X is given below :

$X = x_i$	1	2	3	4	5
$P(X = x_i)$	k	2k	3k	4k	5k

Find the value of k and the mean and variance of X.