

Total No. of Questions : 24  
Total No. of Printed Pages : 3

Regd.  
No.

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Part - III

**MATHEMATICS - PAPER - II (B)**

(English Version)



Time : 3 Hours

Max. Marks : 75

Note : This question paper consists of three Sections - A, B and C.

**SECTION - A**

10x2=20

I. Very short answer type questions :

(i) Answer *all* the questions.

(ii) Each question carries *two* marks.

- If the length of the tangent from (2, 5) to the circle  $x^2 + y^2 - 5x + 4y + k = 0$  is  $\sqrt{37}$  then find k.
- Find the equation of the circle whose end points of a diameter are (1, 2), (4, 6).
- Find the equation of the common chord of the circles  $x^2 + y^2 - 4x - 4y + 3 = 0$ ,  $x^2 + y^2 - 5x - 6y + 4 = 0$ .
- Find the value of k if the line  $2y = 5x + k$  is a tangent to the parabola  $y^2 = 6x$ .
- If the eccentricity of a hyperbola is  $\frac{5}{4}$ , then find the eccentricity of its conjugate hyperbola.

6. Evaluate  $\int \frac{1}{7x+3} dx$  on  $I \subset \mathbf{R} \setminus \left\{-\frac{3}{7}\right\}$ .

7. Evaluate  $\int \sqrt{1 - \sin 2x} dx$  on  $I \subset \left[2n\pi - \frac{3\pi}{4}, 2n\pi + \frac{\pi}{4}\right]$ ,  $n \in \mathbf{Z}$ .

8. Evaluate  $\int_0^4 |2 - x| dx$ .

9. Find  $\int_0^{\frac{\pi}{2}} \sin^{10} x \, dx$ .

10. Find the order and degree of the differential equation  $\frac{d^2y}{dx^2} = \left[ 1 + \left( \frac{dy}{dx} \right)^2 \right]^{\frac{5}{3}}$ .

### SECTION - B

II. Short answer type questions :

5x4=20

- (i) Answer *any five* questions.
- (ii) Each question carries *four* marks.
11. Find the length of the chord intercepted by the circle  $x^2 + y^2 - 8x - 2y - 8 = 0$  on the line  $x + y + 1 = 0$ .
12. Find the radical centre of the following circles.  
 $x^2 + y^2 - 4x - 6y + 5 = 0$ ,  $x^2 + y^2 - 2x - 4y - 1 = 0$ ,  $x^2 + y^2 - 6x - 2y = 0$
13. Find the length of major axis, minor axis, latus rectum, eccentricity, coordinates of centre, foci and the equations of directrices of the ellipse  $9x^2 + 16y^2 = 144$ . <https://www.apboardonline.com>
14. If the normal at one end of a latus rectum of the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  passes through one end of the minor axis, then show that  $e^4 + e^2 = 1$  ( $e$  is the eccentricity of the ellipse).
15. Find the equations of the tangents to the hyperbola  $x^2 - 4y^2 = 4$  which are (i) parallel (ii) perpendicular to the line  $x + 2y = 0$ .
16. Evaluate  $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} \, dx$ .
17. Solve  $\frac{dy}{dx} - x \tan(y - x) = 1$ .

## SECTION - C

5x7=35

Long answer type questions :

- (i) Answer any five questions.
- (ii) Each question carries seven marks.
18. Show that the circles  $x^2 + y^2 - 6x - 2y + 1 = 0$ ,  $x^2 + y^2 + 2x - 8y + 13 = 0$  touch each other. Find the point of contact and the equation of common tangent at their point of contact.
19. If  $(2, 0)$ ,  $(0, 1)$ ,  $(4, 5)$  and  $(0, c)$  are concyclic then find  $c$ .
20. Prove that the area of the triangle inscribed in the parabola  $y^2 = 4ax$  is  $\frac{1}{8a} |(y_1 - y_2)(y_2 - y_3)(y_3 - y_1)|$  sq. units where  $y_1, y_2, y_3$  are the ordinates of its vertices.
21. Obtain the reduction formula for  $\int \sin^n x dx$  for an integer  $n \geq 2$  and deduce  $\int \sin^4 x dx$ .
22. Evaluate  $\int \frac{2 \sin x + 3 \cos x + 4}{3 \sin x + 4 \cos x + 5} dx$ .
23. Evaluate  $\int_0^1 \frac{\log(1+x)}{1+x^2} dx$ .
24. Solve  $\sqrt{1+x^2} \sqrt{1+y^2} dx + xy dy = 0$ .