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Total No. of Questions—**24**

Total No. of Printed Pages—**4** **Regd. No.** | | | | | | | | | | |

Part III

MATHEMATICS

Paper II(B)

(English Version)

Time : 3 Hours ***Max. Marks : 75***

Note :—This question paper consists of THREE sections A, B and C.

SECTION A 10×2=20

I. Very Short Answer Type Questions :

(*i*) Attempt ALL questions.

(*ii*) Each question carries TWO marks.

1. Obtain the parametric equation of circle $(x - 3)^2 + (y - 4)^2 = 8^2$.

2. Find the equation of the normal at P(3, 5) of the circle $S \equiv x^2 + y^2 - 10x - 2y + 6$.

3. If $x^2 + y^2 - 5x - 14y - 34 = 0$, $x^2 + y^2 + 2x + 4y + k = 0$ circles are orthogonal, then find 'k'.

4. Find the value of k if the line $2y = 5x + k$ is a tangent to the parabola $y^2 = 6x$.

5. Find the eccentricity and length of the latus rectum of the hyperbola $16y^2 - 9x^2 = 144$.

6. Evaluate :

$$\int \frac{\sin(\tan^{-1} x)}{1 + x^2} dx, \; x \in \mathbf{R}.$$

7. Evaluate :

$$\int e^x \left[\frac{1 + x \log x}{x} \right] dx \text{ on } (0, \infty).$$

8. Evaluate :

$$\int_0^{\pi} \sqrt{2 + 2 \cos \theta} \, d\theta.$$

9. Find :

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^2 x \cos^4 x \, dx.$$

10. Find the order and degree of :

$$x^{1/2} \left[\frac{d^2y}{dx^2} \right]^{1/3} + x \frac{dy}{dx} + y = 0.$$

SECTION B

5×4=20

II. Short Answer Type Questions :

(*i*) Attempt ANY FIVE questions.

(*ii*) Each question carries FOUR marks.

11. Find the length of the chord intercepted by the circle $x^2 + y^2 - x + 3y - 22 = 0$ on the line $y = x - 3$.

12. Find the radical centre of the three circles :

(*i*) $x^2 + y^2 - 4x - 6y + 5 = 0$

(*ii*) $x^2 + y^2 - 2x - 4y - 1 = 0$

(*iii*) $x^2 + y^2 - 6x - 2y = 0$.

13. Find the equation of the ellipse in the standard form whose distance between foci is 2 and the length of latus rectum is $\frac{15}{2}$.

14. If a tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ $(a > b)$ meets its major axis and minor axis at M and N respectively, then prove that $\frac{a^2}{(CM)^2} + \frac{b^2}{(CN)^2} = 1$ where C is the centre of the ellipse.

15. Find the centre, foci, eccentricity, equation of the directrices, length of the latus rectum of the hyperbola $x^2 - 4y^2 = 4$.

16. Find the area bounded by the curves $y = \sin x$ and $y = \cos x$ between any *two* consecutive points of intersection.

17. Solve :

$$(1 + x^2) \frac{dy}{dx} + y = e^{\tan^{-1} x}.$$

SECTION C

5×7=35

III. Long Answer Type Questions :

(*i*) Attempt ANY FIVE questions.

(*ii*) Each question carries SEVEN marks.

18. Find the equation of circle passing through the three points (3, 4), (3, 2), (1, 4).

19. Find the direct common tangents of the circles $x^2 + y^2 + 22x - 4y - 100 = 0$ and $x^2 + y^2 - 22x + 4y + 100 = 0$.

20. Prove that the area of the triangle formed by the tangents at (x_1, y_1), (x_2, y_2) and (x_3, y_3) to the parabola $y^2 = 4ax$ $(a > 0)$ is $\frac{1}{16a} |(y_1 - y_2)(y_2 - y_3)(y_3 - y_1)|$ sq. units.

21. Evaluate :

$$\int \frac{2 \cos x + 3 \sin x}{4 \cos x + 5 \sin x} dx.$$

22. Obtain reduction formula for $I_n = \int \tan^n x \, dx$, n being a positive integer $n \geq 2$ and deduce the value of $\int \tan^6 x \, dx$.

23. Show that :

$$\int_0^{\frac{\pi}{2}} \frac{x}{\sin x + \cos x} dx = \frac{\pi}{2\sqrt{2}} \log(\sqrt{2} + 1)$$

24. Solve :

$$(x^2y - 2xy^2)dx = (x^3 - 3x^2y)dy .$$