

Total No. of Questions—24

Total No. of Printed Pages—4

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Part III

MATHEMATICS

Paper II(A)

(English Version)

Time : 3 Hours

Max. Marks : 75

Note :—This question paper consists of THREE sections A, B and C.

SECTION A

10×2=20

I. Very Short Answer Type Questions :

(i) Answer ALL questions.

(ii) Each question carries TWO marks.

- Find the complex conjugate of $(2 + 5i)(-4 + 6i)$.
- If $x + iy = \text{cis } \alpha \cdot \text{cis } \beta$, then find the value of $x^2 + y^2$.
- If ABC are angles of a triangle such that $x = \text{cis } A$, $y = \text{cis } B$, $z = \text{cis } C$, then find the value of xyz .
- For what values of x the expression $x^2 - 5x - 14$ is positive ?
- If 1, 1, α are the roots of $x^3 - 6x^2 + 9x - 4 = 0$, then find α .
- Find the number of ways of arranging the letter of the word "MATHEMATICS".
- Find the value of ${}^{10}C_5 + {}^{2 \cdot 10}C_4 + {}^{10}C_3$.
- Find the 8th term of $\left(1 - \frac{5x}{2}\right)^{-3/5}$.
- Find the mean deviation about the mean for the following data :
3, 6, 10, 4, 9, 10.

10. If the mean and variance of a binomial variable X are 2.4 and 1.44 respectively, find the parameters of the distribution X . (Binomial).

SECTION B

5×4=20

II. Short Answer Type Questions :

- (i) Answer ANY FIVE questions.
(ii) Each question carries FOUR marks.
11. If $(x - iy)^{1/3} = a - ib$, then show that :

$$\frac{x}{a} + \frac{y}{b} = 4(a^2 - b^2).$$

12. Find the maximum value of the function $\frac{x^2 + 14x + 9}{x^2 + 2x + 3}$ over $\mathbf{R}$.
13. Find the sum of all 4-digit numbers that can be formed using the digits 1, 3, 5, 7, 9.
14. Prove that :

$${}^{25}C_4 + \sum_{r=0}^4 ({}^{29-r}C_3) = {}^{30}C_4.$$

15. Resolve $\frac{x^2 + 5x + 7}{(x - 3)^3}$ into partial fractions.
16. A and B are events with $P(A) = 0.5$, $P(B) = 0.4$ and $P(A \cap B) = 0.3$. Find the probability that :
- (i) A does not occur
(ii) Neither A nor B occurs.
17. A problem in calculus is given to two students A and B whose chances of solving it are $\frac{1}{3}$ and $\frac{1}{4}$ respectively. Find the probability of the problem being solved if both of them try independently.

SECTION C

5×7=35

III. Long Answer Type Questions :

- (i) Answer ANY FIVE questions.
(ii) Each question carries SEVEN marks.

18. If n is a positive integer, show that :

$$(P + iQ)^{\frac{1}{n}} + (P - iQ)^{\frac{1}{n}} = 2(P^2 + Q^2)^{\frac{1}{2n}} \cdot \cos \left[\frac{1}{n} \tan^{-1} \frac{Q}{P} \right].$$

19. Solve the equation :

$$6x^6 - 25x^5 + 31x^4 - 31x^2 + 25x - 6 = 0.$$

20. For $r = 0, 1, 2, \dots, n$, prove that :

$C_0C_r + C_1C_{r+1} + C_2C_{r+2} + \dots + C_{n-r} \cdot C_n = {}^{2n}C_{(n+r)}$ and hence deduce that :

(i) $C_0^2 + C_1^2 + C_2^2 + \dots + C_n^2 = {}^{2n}C_n$

(ii) $C_0C_1 + C_1C_2 + C_2C_3 + \dots + C_{n-1} \cdot C_n = {}^{2n}C_{n+1}$.

21. Find the sum to infinite terms of the series :

$$\frac{7}{5} \left(1 + \frac{1}{10^2} + \frac{1.3}{1.2} \cdot \frac{1}{10^4} + \frac{1.3.5}{1.2.3} \cdot \frac{1}{10^6} + \dots \infty \right)$$

22. Find the mean deviation from the mean of the following data, using the step deviation method :

Marks	No. of Students
0—10	6
10—20	5
20—30	8
30—40	15
40—50	7
50—60	6
60—70	3

23. (a) State and prove Addition theorem on probability.
- (b) Find the probability of drawing an ace or a spade from a well shuffled pack of 52.
24. If X is a random variable with probability distribution $P(X = k) = \frac{(k+1)C}{2^k}$, $k = 0, 1, 2, 3, \dots$, then find C .