166

II

Total No. of Questions – 24
Total No. of Printed Pages – 4

Regd. No. | | | | | | | | | | |

Part – III
MATHEMATICS, Paper – I (A)
(English Version)

Time : 3 Hours] **[Max. Marks : 75**

Note : This question paper consists of **three** Sections – **A, B** and **C.**

SECTION – A

10 × 2 = 20

I. Very Short Answer Type questions :

(i) Answer **all** the questions.

(ii) Each question carries **two** marks.

1. If $f(x) = 2x - 1$, $g(x) = \frac{x+1}{2}$ then find (gof) (x).

2. If $f = \{(1, 2) (2, -3), (3, -1)\}$ then find (i) 2f (ii) f^2.

3. Find the trace of the matrix $A = \begin{bmatrix} 1 & 3 & -5 \\ 2 & -1 & 5 \\ 2 & 0 & 1 \end{bmatrix}$.

4. Find the rank of the matrix $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 2 & 1 \end{bmatrix}$.

5. If $\bar{a} = 2\bar{i} + 5\bar{j} + \bar{k}$ and $\bar{b} = 4\bar{i} + m\bar{j} + n\bar{k}$ are collinear vectors then find m, n.

6. Find the vector equation of the plane passing through the points (0, 0, 0), (0, 5, 0) and (2, 0, 1).

7. Find the angle between the planes $\bar{r} \cdot (2\bar{i} - \bar{j} + 2\bar{k}) = 3$ and $\bar{r} \cdot (3\bar{i} + 6\bar{j} + \bar{k}) = 4$.

8. If $\tan 20° = \lambda$ then show that $\frac{\tan 160° - \tan 110°}{1 + \tan 160° \tan 110°} = \frac{1 - \lambda^2}{2\lambda}$.

9. Find the range of $7 \cos x - 24 \sin x + 5$.

10. Prove that $(\cosh x - \sinh x)^n = \cosh(nx) - \sinh(nx)$.

SECTION – B

$5 \times 4 = 20$

II. Short Answer Type questions :

(i) Answer any **five** questions.

(ii) Each question carries **four** marks.

11. If $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$ then show that $AA' = A'A = I$.

12. If the points whose position vectors are $3\bar{i} - 2\bar{j} - \bar{k}$, $2\bar{i} + 3\bar{j} - 4\bar{k}$, $-\bar{i} + \bar{j} + 2\bar{k}$ and $4\bar{i} + 5\bar{j} + \lambda\bar{k}$ are coplanar then show that $\lambda = \frac{-146}{17}$.

13. Find the vector area and area of the parallelogram having $\bar{a} = \bar{i} + 2\bar{j} - \bar{k}$ and $\bar{b} = 2\bar{i} - \bar{j} + 2\bar{k}$ as adjacent sides.

14. Prove that $\sin^4\frac{\pi}{8} + \sin^4\frac{3\pi}{8} + \sin^4\frac{5\pi}{8} + \sin^4\frac{7\pi}{8} = \frac{3}{2}$.

15. Solve $7\sin^2\theta + 3\cos^2\theta = 4$.

16. Prove that $\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{8} = \frac{\pi}{4}$.

17. If $\cot\frac{A}{2}, \cot\frac{B}{2}, \cot\frac{C}{2}$ are in A.P. then prove that **a, b, c are in A.P.**

SECTION – C

5 × 7 = 35

III. Long Answer Type questions :

(i) Answer any **five** questions.

(ii) Each question carries **seven marks.**

18. (a) If $f(x) = \frac{x+1}{x-1}, (x \neq \pm 1)$ then find (fofof) (x).

(b) If $f : A \to B,\ g : B \to C,\ h : C \to D$ are functions then show that ho(gof) = (hog)of.

19. Show that $49^n + 16n - 1$ is divisible by 64 for all positive integers by using Mathematical induction.

20. Show that $\begin{vmatrix} a^2+2a & 2a+1 & 1 \\ 2a+1 & a+2 & 1 \\ 3 & 3 & 1 \end{vmatrix} = (a-1)^3$.

21. Solve the equations using Cramer's Rule $x + y + z = 1$, $2x + 2y + 3z = 6$, $x + 4y + 9z = 3$.

22. Find the shortest distance between the skew lines

$\bar{r} = (6\bar{i} + 2\bar{j} + 2\bar{k}) + t\,(\bar{i} - 2\bar{j} + 2\bar{k})$,

and $\bar{r} = (-4\bar{i} - \bar{k}) + s\,(3\bar{i} - 2\bar{j} - 2\bar{k})$.

23. If $A + B + C = \frac{\pi}{2}$ then prove that $\cos 2A + \cos 2B + \cos 2C = 1 + 4 \sin A \sin B \sin C$.

24. Prove that $r + r_3 + r_1 - r_2 = 4R \cos B$.
