# 197

**B.C.**

Total No. of Questions—**15**

Total No. of Printed Pages—**3** **Regd. No.** | | | | | | | | | | |

**Part III**

**MATHEMATICS**

**(Bridge Course) (For B.P.C. Candidates)**

**Paper I**

**(English Version)**

***Time : 3 Hours*** ***Max. Marks : 75***

***Note*** **:—** This question paper consists of TWO Sections A and B.

**SECTION–A** 10×3=30

**I.** Short answer type questions :

(*i*) Answer ALL the questions.

(*ii*) Each question carries THREE marks.

**1.** If $A = \begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix}$, then show that $A^2 = -I$.

**2.** Find the unit vector in the direction of the vector $\bar{a} = 2\bar{i} + 3\bar{j} + \bar{k}$.

**3.** If $\bar{a} = 2\bar{i} + 2\bar{j} - 3\bar{k}$, $\bar{b} = 3\bar{i} - \bar{j} + 2\bar{k}$, then find the angle between $2\bar{a} + \bar{b}$ and $\bar{a} + 2\bar{b}$.

**4.** Find the maximum and minimum values of :

$$\cos\left(x + \frac{\pi}{3}\right) + 2\sqrt{2}\sin\left(x + \frac{\pi}{3}\right) - 3.$$

**5.** Evaluate :

$$\sin^2 82\frac{1}{2}^\circ - \sin^2 22\frac{1}{2}^\circ.$$

**6.** Find the equation of the straight line which makes an angle $135°$ with the positive $x$-axis and passing through the point $(3, -2)$.

**7.** Find the point of intersection of the straight lines $4x + 8y - 1 = 0$ and $2x - y + 1 = 0$.

**8.** Find '$x$' if the distance between the points $(5, -1, 7)$ and $(x, 5, 7)$ is '9' units.

**9.** Evaluate :

$$\lim_{x \to 0} \left[ \frac{e^x - \sin x - 1}{x} \right].$$

**10.** If $f(x) = 2x^2 + 3x - 5$, then prove that $f'(0) + 3 \cdot f'(-1) = 0$.

## SECTION-B

$3 \times 15 = 45$

**II.** Long answer type questions :

(*i*) Attempt ANY THREE questions.

(*ii*) Each question carries FIFTEEN marks.

**11.** (*a*) Solve :

$$x + y + z = 1$$
$$2x + 2y + 3z = 6$$
$$x + 4y + 9z = 3$$

by matrix-inversion method.

(*b*) Show that :

$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix} = abc(a - b)(b - c)(c - a).$$

**12.** (*a*) If the points whose position vectors are $3\bar{i} - 2\bar{j} - \bar{k}$, $2\bar{i} + 3\bar{j} - 4\bar{k}$, $-\bar{i} + \bar{j} + 2\bar{k}$ and $4\bar{i} + 5\bar{j} + \lambda\bar{k}$ are coplanar, then show that $\lambda = \dfrac{-146}{17}$.

(*b*) Prove that for any three vectors $\bar{a}$, $\bar{b}$ and $\bar{c}$, $\left[\bar{b} + \bar{c} \quad \bar{c} + \bar{a} \quad \bar{a} + \bar{b}\right] = 2\left[\bar{a} \quad \bar{b} \quad \bar{c}\right]$.

**13.** (*a*) Prove that :

$$\sin^4\left(\frac{\pi}{8}\right) + \sin^4\left(\frac{3\pi}{8}\right) + \sin^4\left(\frac{5\pi}{8}\right) + \sin^4\left(\frac{7\pi}{8}\right) = \frac{3}{2}.$$

(*b*) If A, B, C are angles of a triangle, then prove that : $\cos 2A + \cos 2B + \cos 2C = -1 - 4\cos A \cos B \cos C$.

**14.** (*a*) Find the circumcenter of the triangle whose sides are given by $3x - y - 5 = 0$, $x + 2y - 4 = 0$ and $5x + 3y + 1 = 0$.

(*b*) Find the value of '$k$' if the angle between the straight lines $4x - y + 7 = 0$ and $kx - 5y - 9 = 0$ is 45°.

**15.** (*a*) If $x^y = e^{x-y}$, then show that :

$$\frac{dy}{dx} = \frac{\log x}{(1 + \log x)^2}.$$

(*b*) Find the equations of the tangent and normal to the curve $y = 5x^4$ at the point (1, 5).