## Graduate Aptitude Test in Engineering 2021 Organising Institute - IIT Bombay

Engineering Mathematics (XE-A)

## General Aptitude (GA)

Q. 1 - Q. 5 Multiple Choice Question (MCQ), carry ONE mark each (for each wrong answer: $-\mathbf{1 / 3}$ ).

| Q.1 | Gauri said that she can play the keyboard___ her sister. |
| ---: | :--- |
| (A) | as well as |
| (B) | as better as |
| (C) | as nicest as |
| (D) | as worse as |


| Q. 2 | A transparent square sheet shown above is folded along the dotted line. The folded sheet will look like $\qquad$ . |
| :---: | :---: |
| (A) |  |
| (B) |  |
| (C) |  |
| (D) |  |

Engineering Mathematics (XE-A)

| Q.3 | If $\boldsymbol{\theta}$ is the angle, in degrees, between the longest diagonal of the cube and <br> any one of the edges of the cube, then, $\cos \boldsymbol{\theta}=$ |
| :--- | :--- |
| (A) | $\frac{1}{2}$ |
| (B) | $\frac{1}{\sqrt{3}}$ |
| (C) | $\frac{1}{\sqrt{2}}$ |
| (D) | $\frac{\sqrt{3}}{2}$ |


| Q. 4 | If $\left(x-\frac{1}{2}\right)^{2}-\left(x-\frac{3}{2}\right)^{2}=x+2$, then the value of $x$ is: |
| ---: | :--- |
| (A) | 2 |
| (B) | 4 |
| (C) | 6 |
| (D) | 8 |


| Q. 5 | Pen : Write :: Knife : <br> Which one of the following options maintains a similar logical relation in the <br> above? |
| ---: | :--- |
| (A) | Vegetables |
| (B) | Sharp |
| (C) | Cut |
| (D) | Blunt |

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Q. 6 - Q. 10 Multiple Choice Question (MCQ), carry TWO marks each (for each wrong answer: $-2 / 3$ ).

| Q.6 | Listening to music during exercise improves exercise performance and <br> reduces discomfort. Scientists researched whether listening to music while <br> studying can help students learn better and the results were inconclusive. <br> Students who needed external stimulation for studying fared worse while <br> students who did not need any external stimulation benefited from music. <br> Which one of the following statements is the CORRECT inference of the <br> above passage? |
| ---: | :--- |
| (A) | Listening to music has no effect on learning and a positive effect on physical <br> exercise. |
| (B) | Listening to music has a clear positive effect both on physical exercise and on <br> learning. |
| (C) | Listening to music has a clear positive effect on physical exercise. Music has a <br> positive effect on learning only in some students. |
| (D) | Listening to music has a clear positive effect on learning in all students. Music <br> has a positive effect only in some students who exercise. |

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Engineering Mathematics (XE-A)


Engineering Mathematics (XE-A)

| Q.8 | The number of students in three classes is in the ratio 3:13:6. If 18 students <br> are added to each class, the ratio changes to 15:35:21. <br> The total number of students in all the three classes in the beginning was: |
| :--- | :--- |
| (A) | 22 |
| (B) | 66 |
| (C) | 88 |
| (D) | 110 |

\begin{tabular}{|c|c|}

\hline Q. 9 \& |  |
| :--- |
| The number of units of a product sold in three different years and the respective net profits are presented in the figure above. The cost/unit in Year 3 was ` 1 , which was half the cost/unit in Year 2. The cost/unit in Year 3 was one-third of the cost/unit in Year 1. Taxes were paid on the selling price at $10 \%, 13 \%$ and $15 \%$ respectively for the three years. Net profit is calculated as the difference between the selling price and the sum of cost and taxes paid in that year. |
| The ratio of the selling price in Year 2 to the selling price in Year 3 is | <br>

\hline A) \& $4: 3$ <br>
\hline (B) \& $1: 1$ <br>
\hline (C) \& 3:4 <br>
\hline (D) \& $1: 2$ <br>
\hline
\end{tabular}

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Engineering Mathematics (XE-A)

| Q.10 | Six students $\mathbf{P}, \mathbf{Q}, \mathbf{R}, \mathbf{S ,}, \mathbf{T}$ and U, with distinct heights, compare their <br> heights and make the following observations. <br> Observation I: $\mathbf{S}$ is taller than $\mathbf{R}$. <br> Observation II: Q is the shortest of all. <br> Observation III: U is taller than only one student. <br> Observation IV: T is taller than S but is not the tallest. <br> The number of students that are taller than $\mathbf{R}$ is the same as the number of <br> students shorter than |
| :--- | :--- |
| (A) | T |
| (B) | R |
| (C) | S |
| (D) | P |

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Engineering Mathematics (XE-A)

## Engineering Mathematics (XE-A)

Q. 1 - Q. 3 Multiple Choice Question (MCQ), carry ONE mark each (for each wrong answer: - 1/3).

| Q. 1 | Let $S=\left\{A X: \quad A=\left[\begin{array}{rr} 2 & -4 \\ 1 & 1 \\ 1 & -1 \end{array}\right] \quad \text { and } X=\left[\begin{array}{l} x_{1} \\ x_{2} \end{array}\right]\right\} .$ <br> If $\left[\begin{array}{r}-1 \\ \alpha \\ 1\end{array}\right] \in S$, then the value of $\alpha$ is |
| :---: | :---: |
| (A) | -4 |
| (B) | -2 |
| (C) | 2 |
| (D) | 4 |


| Q. 2 | Let $\boldsymbol{C}$ be the boundary of the region $\boldsymbol{R}: \mathbf{0} \leq \boldsymbol{x} \leq \boldsymbol{\pi}, \mathbf{0} \leq \boldsymbol{y} \leq \sin \boldsymbol{x}$ in the <br> $\boldsymbol{x} \boldsymbol{y}$-plane and $\boldsymbol{\alpha}$ be the area of the region $\boldsymbol{R}$. If $\boldsymbol{C}$ traverses once in the counter <br> clockwise direction, then the value of the line integral $\oint_{\boldsymbol{C}}(\mathbf{y ~} \boldsymbol{d} \boldsymbol{x}+5 \boldsymbol{x} \boldsymbol{d} \boldsymbol{y})$ <br> is equal to |
| :--- | :--- |
| (A) | $\alpha$ |
| (B) | $2 \alpha$ |
| (C) | $3 \alpha$ |
| (D) | $4 \alpha$ |

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Engineering Mathematics (XE-A)

| Q. 3 | Given that $i=\sqrt{-1}$. The value of $\lim _{z \rightarrow e^{\frac{\pi i}{3}}} \frac{z^{3}+\mathbf{1}}{z^{4}+z^{2}+1}$ <br> is |
| :---: | :---: |
| (A) | $\frac{3}{4}+i \frac{\sqrt{3}}{4}$ |
| (B) | $\frac{3}{4}-i \frac{\sqrt{3}}{4}$ |
| (C) | $\frac{-3}{4}+i \frac{\sqrt{3}}{4}$ |
| (D) | $\frac{-3}{4}-i \frac{\sqrt{3}}{4}$ |

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Engineering Mathematics (XE-A)
Q. 4 - Q. 7 Numerical Answer Type (NAT), carry ONE mark each (no negative marks).

| Q. 4 | Let $f(x)$ be a non-negative continuous function of real variable $x$. If the area <br> under the curve $y=f(x)$ from $x=0$ to $x=a$ is $\frac{a^{2}}{2}+\frac{a}{2} \sin a+\frac{\pi}{2} \cos a-\frac{\pi}{2}$, <br> then the value of $f\left(\frac{\pi}{2}\right)$ is__ (round off to one decimal <br> place). |
| :--- | :--- |


| Q. 5 | If the numerical approximation of the value of the integral $\int_{0}^{4} 2^{\alpha x} d x$ using <br> the Trapezoidal rule with two subintervals is 9, then the value of the real <br> constant $\alpha$ is <br> (round off to one decimal place). |
| :--- | :--- |


| Q. 6 | Let the transformation $y(x)=e^{x} v(x)$ reduce the ordinary differential <br> equation |
| :--- | :--- | :--- |
| $\qquad$$x \frac{d^{2} y}{d x^{2}}+2(1-x) \frac{d y}{d x}+(x-2) y=0 ; x>0$ |  |
| to$\alpha x \frac{d^{2} v}{d x^{2}}+2 \beta \frac{d v}{d x}+3 \gamma v=0$, <br> where $\alpha, \beta, \gamma$ are real constants. Then, the arithmetic mean of $\alpha, \beta, \gamma$ <br> is (round off to three decimal places). |  |


| Q. 7 | A person, who speaks the truth 3 out of 4 times, throws a fair dice with six <br> faces and informs that the outcome is 5. The probability that the outcome is <br> really 5 is <br> (round off to three decimal places). |
| :--- | :--- |

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Engineering Mathematics (XE-A)
Q. 8 - Q. 9 Multiple Choice Question (MCQ), carry TWO marks each (for each wrong answer: $-2 / 3$ ).

| Q. 8 | Let $\boldsymbol{f}(\boldsymbol{x}, \boldsymbol{y})=\boldsymbol{x}^{4}+\boldsymbol{y}^{4}-2 \boldsymbol{x}^{2}+\mathbf{4 x y}-\mathbf{2} \boldsymbol{y}^{2}+\boldsymbol{\alpha}$ be a real valued function. <br> Then, which one of the following statements is TRUE for all $\boldsymbol{\alpha} \boldsymbol{?}$ |
| ---: | :--- |
| (A) | $(0,0)$ is not a stationary point of $f$ |
| (B) | $f$ has a local maxima at $(0,0)$ |
| (C) | $f$ has a local minima at $(0,0)$ |
| (D) | $f$ has a saddle point at $(0,0)$ |


| Q. 9 | Let $\boldsymbol{u}(\boldsymbol{x}, \boldsymbol{y})=\left(\boldsymbol{x}^{2}-\boldsymbol{y}^{2}\right) \boldsymbol{v}(\boldsymbol{x}, \boldsymbol{y})$ be such that both $\boldsymbol{u}(\boldsymbol{x}, \boldsymbol{y})$ and $\boldsymbol{v}(\boldsymbol{x}, \boldsymbol{y})$ satisfy <br> the Laplace equation in a domain $\boldsymbol{\Omega}$ of the $\boldsymbol{x y}$-plane. Then, which one of the <br> following is TRUE in $\boldsymbol{\Omega}$ ? |
| :--- | :--- |
| (A) | $x \frac{\partial v}{\partial x}-y \frac{\partial v}{\partial y}=0$ |
| (B) | $x \frac{\partial v}{\partial x}+y \frac{\partial v}{\partial y}=0$ |
| (C) | $x \frac{\partial v}{\partial y}-y \frac{\partial v}{\partial x}=0$ |
| (D) | $x \frac{\partial v}{\partial y}+y \frac{\partial v}{\partial x}=0$ |

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Engineering Mathematics (XE-A)
Q. 10 - Q. 11 Numerical Answer Type (NAT), carry TWO marks each (no negative marks).
Q. 10 Let $I$ denote the identity matrix of order 7 , and $A$ be a $7 \times 7$ real matrix having characteristic polynomial $C_{A}(\lambda)=\lambda^{2}(\lambda-1)^{\alpha}(\lambda+2)^{\beta}$, where $\alpha$ and $\beta$ are positive integers. If $A$ is diagonalizable and $\operatorname{rank}(A)=\operatorname{rank}(A+$ $2 I)$, then $\operatorname{rank}(A-I)$ is $\qquad$ (in integer).
Q. 11

Let $C_{1}$ be the line segment from $(0,1)$ to $\left(\frac{4}{5}, \frac{3}{5}\right)$, and let $C_{2}$ be the arc of the circle $x^{2}+y^{2}=1$ from $(0,1)$ to $\left(\frac{4}{5}, \frac{3}{5}\right)$. If

$$
\alpha=\int_{C_{1}}\left(\frac{2 x}{y} \hat{\imath}+\frac{1-x^{2}}{y^{2}} \hat{\jmath}\right) \cdot d \vec{r} \text { and } \beta=\int_{C_{2}}\left(\frac{2 x}{y} \hat{\imath}+\frac{1-x^{2}}{y^{2}} \hat{\jmath}\right) \cdot d \vec{r}
$$

where $\vec{r}=x \hat{\imath}+y \hat{\jmath}$, then the value of $\alpha^{2}+\beta^{2}$ is $\qquad$ (round off to two decimal places).

END OF THE QUESTION PAPER

